Fuzzy Models Of Measurement Uncertainty And Their Application

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Abstract

The classical methods of measurement uncertainty estimation are based on probability theory. In [1] these methods are divided into three groups:
- analytical: by use of convolution, with use of characteristic functions;
- numerical: Monte-Carlo method [2], discrete convolution;
- approximate methods, including GUM-method [3] and application of the moments of distributions.

It is reasonable to add a group of fuzzy methods (fig. 1). They are connected with possibility theory established by Zadeh (1978). Both theories have common feature – they are based on set functions. These are functions of probability density and membership functions. Connection between the theories is provided by so-called probability–possibility transformation. Techniques of discrete and continuous transformations are offered in [4, 5, 6]. This work is devoted to consideration of fuzzy models of measurement uncertainty and their application. The example about wheat classification is given.

1. Fuzzy representation of the measurement uncertainty components and tools for combining

The overview of fuzzy models is done in recent paper [9]: p-box, triangular fuzzy number, fuzzy numbers with pseudo-triangular distribution (tpd), L-R fuzzy numbers. The connection of interval models with fuzzy models of measurement uncertainty is established. It is possible to receive possibility distribution for any unimodal (and antimodal) probability density function. The upper-bound estimate is the triangular fuzzy number at representation of measurement uncertainty.

Composition of two fuzzy numbers could be realized in continuous form operating by the means of L-R fuzzy numbers or in discrete form, that is appropriate for computer usage version. This process includes application of operation on fuzzy sets, first of all triangular norms.

Uncertainty

Measured quantity value

Measurement result

Representation

Probability theory

Possibility theory

Evidence theory

probability density function $p(x)$

membership function $\mu(x)$

Fuzzy numbers with pseudo-triangular distribution (tpd), L-R fuzzy numbers.

Certain classical distribution

Generalized distribution

$gh$-distribution

GLD...

Fig.1. Measurement result and uncertainty representation

The most used parametric classes of triangular norms are: product t-norm; Yager’s t-norm, proposed by Urbanski; Hamacher t-norm; Frank t-norms; Sugeno-Webster t-norms.

All of them were tested. As the effective method in the view point of simplicity and accuracy combination, the product t-norm seems suitable, according the fact, that it is non-parametric operation.
It is known that this fuzzy approach is suitable for one-modal distributions only, but as it was, in the district \(\alpha=0.05\ (P=0.95)\) results of convolution are close comparing with Monte-Carlo simulation (MCS). This fact makes applicable method for these cases too.

There is very important feature – possibility distribution could account nonlinearity bias [8]: that function of density of probability is asymmetrical for nonlinear measurement models, therefore mathematical expectation of distribution does not coincide with midrange. So, this approach could be used for measurement models with significant nonlinearity to obtain coverage interval with nonsymmetrical limits about the expectation.

**2. Generalized distribution usage**

In the current work generalized possibility distributions are proposed.

Using family of probability distributions, for instance symmetrical, we can obtain correspondent family of possibility distributions. For family of probability distributions based on transformation random variable with standard uniform distribution \(U\) by function:

\[
Y = F(U, \lambda),
\]

where \(\lambda\) - shape parameter, correspondent symmetrical fuzzy number could be represented by membership function:

\[
\mu(Y) = \begin{cases} 
2F^{-1}, & U < 0.5 \\
2(1-F^{-1}), & U > 0.5 
\end{cases}
\]

(1)

This approach gives the simplest way to simulate random variable by sources of possibility distribution and inverse function method [2].

Connection of representation ways is descriptive for usage of quantile functions (qf). Approaches to the measurement results representation in quantile domain is summarized in publication [10]. There is a table of basic form \(S(p)\) for classical pdf and for quantile-domain families such as

\[
Q(p) = \lambda_1 + \lambda_2 S(p),
\]

where \(\lambda_1, \lambda_2\) - parameters of location and scale.

Representation of the fuzzy number through qf is corresponded to so-called L-R fuzzy number[11]. This is functions with L- and R-form and “quadruple” of location and scale parameters.

Let's consider example of generalized distribution application - Lambda distribution of Tukey. The family of distributions offered in 1949 [12] based on transformation of random variable with standard uniform distribution \(U\):

\[
X = \begin{cases} 
U^{\lambda} - (1-U)^{\lambda}, & \lambda \neq 0 \\
\log\left(\frac{U}{1-U}\right), & \lambda = 0. 
\end{cases}
\]

(2)

Results of modeling and theirs approximations give equation (3) for evaluation of shape-parameter \(\lambda\) by kurtosis \(\varepsilon\) of uncertainty distribution:

\[
\lambda = \frac{-0.145 \cdot \varepsilon^2 + 0.87 \cdot \varepsilon - 1.07}{\varepsilon^2 - 3.10 \cdot \varepsilon + 2.37}
\]

Some examples of fuzzy numbers for basic pdf representation are given in table. 1.

Such generalized models provide convolution of components by source of high order moments usage, for symmetrical distributions – by kurtosis combining.

**3. Place of fuzzy representation of the measurement result in conformity assessment and intelligence systems**

It is convenient representation of uncertainty in conformity assessment. Many kinds of measurements are carried out to assess compliance with a specification, for example in the control contaminants in food or to determine the class of product, with its corresponded price. So fuzzy representation of the measurement uncertainty is the good tool for risk analysis.

The simplest example is evaluating the quality of grain: «wrong» decision making and identification of food wheat in the feed leads to greater financial losses manufacturer.

Consider a specific example - evaluating the quality of durum wheat (i.e. belong to the definite class). According to normative documents it is provided by a number of requirements for durum wheat (mass fraction of protein, falling number, humidity, etc.). These data is presented in the form of limits. Inconsistencies in at least one parameter makes the product lead in the class below. That is an example, when to apply integrated assessment is impossible.

Let the first controlled setting - bushel weight (a mass of one liter of grain in grams). For example, an absolute error of weight per bushel tester – \(\pm 4\ g\). Because no information about the distribution, we use uniform distribution for uncertainty of type B evaluation. Making of 6 observations gives sample deviation \(S = 2\ g\).

Application of analyzed in [9] simple Product t-norm gives fuzzy number presented on fig. 2.

On the base of this possibility distribution function decision about category of wheat should be made.
Examples of fuzzy numbers for basic pdf

<table>
<thead>
<tr>
<th>PDF by Monte-Carlo modeling using $\lambda$-distribution</th>
<th>Gauss $\varepsilon = 3$</th>
<th>Uniform $\varepsilon = 1.8$</th>
<th>Student($v=10$) $\varepsilon = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuzzy number using $\lambda$-distribution</td>
<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Graph" /></td>
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</tbody>
</table>

Fig.2. Fuzzy models application in wheat classification

Decision about the third category of wheat is made with level of confidence (certainty factor) $CF=1-0.15=0.85$. In order to assess the probability of the product classification, all parameters can be represented in a classification procedure rules, where there are several prerequisites:

1. When bushel weight the i-th class (certainty factor $CF_i$) and
2. If the mass fraction of protein corresponds to i-th class (certainty factor CF2) and m. if ... ... (certainty factor CFm).

Certainty factor of conclusion (conditions combined by modifier «AND») is determined by the formula (3.4):

\[ CF = \min(CF_1, CF_2, ..., CF_m) \]

Thus, the result of classification of wheat to the i-th class is accompanied by a confidence coefficient equal joint confidence coefficient conditions. For example, let a consolidated Table 3.13 Classification results for individual properties. Decided in favor of a lower class, but the total value of probability must satisfy both interested parties (seller and buyer), that may be installed base of its value below which the probability value is considered unacceptable.

2. Conclusions

The overview of fuzzy models is done: p-box, triangular fuzzy number, fuzzy numbers with pseudo-triangular distribution (tpd) and generalized distributions.

Different types of triangular norms are considered for purposes of composition distribution obtaining. Using the simple product t-norm, new method for expanded uncertainty (coverage interval) is proposed.

Generalized distributions provide component convolution by combined kurtosis evaluation.

This method could be applied for models with significant nonlinearity, that is the restriction of GUM, but only for independent components.

Fuzzy representation is the good initial data for decision making and risk analysis.

Bibliography


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