MAKING DECISIONS IN A FUZZY ENVIRONMENT: THE EXAMPLE OF THE OPTIMIZATION TASK OF INVESTMENT IN A COMMUNE

PODEJMOWAŃ DECYZJI W ŚRODOWISKU ROZMYTYM NA PRZYKŁADZIE ZADANIA OPTYMALIZACJI INWESTYCJI W GMINIE

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Abstract

The article deals with a methodology of formation of the fuzzy model supporting decision making. It presents formulation of fuzzy goals and fuzzy constraints, as well as their aggregation.

1. Methodology of formation of the fuzzy model supporting decision making

Being faced with uncertain, incomplete data, imprecise evaluations, definitions or relations, it is very difficult to make an expert decision. A general model of decision making in fuzzy conditions has been presented in papers [1, 2, 3]. This model, apart from any technical applications, may be used for so-called "soft" tasks e.g. socio-economic ones, environmental ones, technological ones as well as medical ones, formulated in linguistic categories.

1.1 Decision making in a fuzzy environment — basic concepts

In Bellman and Zadeh’s [1], as well as Kacprzyk’s [2, 3] approach, the process of decision making takes place in a fuzzy environment, to which contribute fuzzy goals, fuzzy constraints and fuzzy decisions

\[ X, G, C, D \]

where:

\[ X = \{x\} \] — means a set of possible decisions, values and procedures in the situation in question,

\[ G \] — fuzzy goal, i.e. a fuzzy set \( G \) in the set of options \( X \), described by the membership function

\[ \mu_G(x) : X \rightarrow [0,1] \]  

\( D \) — is a fuzzy decision.

The fuzzy decision is a fuzzy set defined in the set of options \( X \) and is the result of aggregation

\[ D = G \ast C \]

what can be presented in the following manner:

\[ \mu_D(x) = \mu_G(x) \ast \mu_C(x) \]

The fuzzy set \( G \) for models supporting decision making should be analysed in categories of (social, decision makers’, etc.) satisfaction. [2]. If the level of satisfaction \( x \) assumes a value which exceeds or is equal to \( \overline{G} \), then \( x \geq \overline{G} \), is \( \mu_G(x) = 1 \), which means full satisfaction from the obtained value \( x \). If \( x \) fails to exceed the value \( \underline{G} \), which is the smallest possible value \( x \), then \( \mu_G(x) = 0 \), which is interpreted as total dissatisfaction. For the values that fulfil the inequality \( \underline{G} < x < \overline{G} \) we receive \( 0 < \mu_G(x) < 1 \), which means that satisfaction from the attained value \( x \) is partial and it grows as it gets closer to the value \( \overline{G} \). The fuzzy constraint \( C \) may be interpreted in a likewise fashion.

The fuzzy decision \( D \) which came into being as a result of the aggregation of two fuzzy sets \( G \) and \( C \) (4) may be defined as "attaining the goal \( G \) and meeting the constraint \( C \)”, so aggregation in this case is equivalent to intersection of two fuzzy sets:

\[ \mu_D(x) = T(\mu_G(x), \mu_C(x)) \]

for each \( x \in X \), where \( T \) - is any t-norm. The simplest definitions are however most commonly
used, e.g. minimum and product, due to the optimization used.

The fuzzy decision type minimum is defined as:
\[
\mu_D(x) = \min(\mu_G(x), \mu_C(x))
\]  
(7)
specifies the conservative attitude of the "safety-first" type.

The fuzzy decision of the product type is specified as:
\[
\mu_D(x) = (\mu_G(x) \cdot \mu_C(x))
\]  
(8)

For minimum and product type decisions it is assumed that the significance of the fuzzy goal and constraint are the same. Otherwise a fuzzy decision type weighted sum is used:
\[
\mu_D(x) = w(\mu_G(x)) + (1-w) \cdot \mu_C(x)
\]  
(9)
where \( w \in [0,1] \). A significance coefficient \( w \) is 0, where only fuzzy constraint fulfillment is important. Where \( w=1 \), however, only fulfillment of the fuzzy goal is important.

In the case where the assigned task is in the form "attain \( G \) or fulfill \( C \)", the fuzzy decision is applied that is equivalent to the sum of two fuzzy sets:
\[
\mu_D(x) = S(\mu_G(x), \mu_C(x))
\]  
(10)
for each \( x \in X \), where \( S \) - is any s-norm, e.g. maximum
\[
\mu_D(x) = \max(\mu_G(x), \mu_C(x))
\]  
(11)
or algebraic sum
\[
\mu_D(x) = \mu_G(x) + \mu_C(x) - \mu_G(x) \mu_C(x)
\]  
(12)
and weighted logical sum
\[
\mu_D(x) = (1 - w_C) \mu_C(x) \lor (1 - w_G) \mu_G(x)
\]  
(13)
where \( \omega_C, \omega_G \leq 1 \).

In the case of many fuzzy goals and constraints the occurrence of \( n > 1 \) fuzzy goals, \( G_1, ..., G_n \), and \( m > 1 \) fuzzy constraints is assumed, \( C_1, ..., C_m \), assuming that all are fuzzy sets, the fuzzy decision may be of the type:
\[
\mu_D(x) = \mu_{G_1}(x) \land ... \land \mu_{G_n}(x) \land \mu_{C_1}(x) \land ... \land \mu_{C_m}(x)
\]  
(14)

In this case, however, fuzzy decisions of the type t-norm, e.g. minimum and product, and s-norm, e.g. maximum, maximum with assigned significance to individual fuzzy goals and constraints are also recognized (15):
\[
\mu_D(x) = w_G \mu_{G_1}(x) \land ... \land w_G \mu_{G_n}(x) \land \mu_{C_1}(x) \land ... \land w_C \mu_{C_m}(x)
\]  
(15)

For all types of fuzzy decisions it is assumed that the optimum decision is a maximizing decision [2]
\[
\mu_D(x^*) = \max_{x \in X} \mu_D(x)
\]  
(16)

1.2. Multi-stage nature of decision making in a fuzzy environment

The process of decision making is multi-stage and involves continuous making, agreeing upon and implementing of multifaceted decisions. The model is presented as a dynamic control system. At the outset we find ourselves in the initial state \( x_0 \in X \) to which a control is applied \( u_0 \in U \) and to which also the fuzzy constraint is imposed \( \mu_{C_1}(u_0) \). A state is reached \( x_1 \in X \) to which the fuzzy goal is imposed \( \mu_{G_1}(x_1) \). Next, a control is applied \( u_1 \) to which the fuzzy constraint is imposed \( \mu_{C_1}(u_1) \) and the fuzzy state is attained \( x_2 \) with the imposed fuzzy goal \( \mu_{G_1}(x_2) \). This process continues at subsequent stages of control until the fuzzy state is attained \( x_N \).

The fuzzy decision \( D(x_0) \) of the multi-stage process of decision making is the result of aggregation of individual fuzzy constraints and goals at subsequent stages of control, which is recorded in the following manner:
\[
D(x_0) = C^0 \ast G^1 \ast ... \ast C^{N-1} \ast G^N
\]  
(17)

Also the decision of the minimum type is used in the multi-stage process of decision making:
\[
\mu_D = (\mu_0, ..., \mu_{N-1} | x_0) = \mu_{C_1}(u_0) \land \mu_{G_1}(x_1) \land ... \land \mu_{C_{N-1}}(u_{N-1}) \land \mu_{G_N}(x_N)
\]  
(18)

2. The exemplary socio-economic fuzzy model of regional development

The fuzzy system supporting managerial decisions is presented as a dynamic control system. The input variables (antecedents of rules) represent the values determining e.g. a quality of life in a given commune such as: \( x_1' \) - transport and communication, \( x_2' \) - municipal services and environmental protection, \( x_3' \) - housing, \( x_4' \) - education, \( x_5' \) - culture and national heritage.
protection, \( x_6^t \) - health care, \( x_7^t \) - welfare, \( x_8^t \) - physical culture and sports.

The data used in the example is taken from Central Statistical Office (GUS) reports [4], they pertain to the Brzeg commune and they are relevant to the period between 2003 and 2007. The model output is a certain indicator \( X_t \) which is a part of the value of individual quality indicators \( X_t = [x_1^t, ..., x_8^t] \) for a given control stage. It concerns planned investment projects and is determined on the basis of expert opinions. It may be calculated by aggregation of unit indicators according to the formula [3]:

\[
\mu_{G_t}(X_t) = \text{AGG}[\mu_{G_1}(x_1^t), ..., \mu_{G_8}(x_8^t)]
\]  

(19)

The total fuzzy quality of life indicator for individual years of planning was calculated based on five types of aggregating functions: a.) minimum, b.) maximum, c.) arithmetic mean, d.) algebraic product, e.) weighted average.

\[ a.) \mu_{G_t}(X_{2003}) = \min[\mu_{G_1}(x_1^{2003}), ..., \mu_{G_8}(x_8^{2003})] = 0.78 \]
\[ b.) \mu_{G_t}(X_{2003}) = \max[\mu_{G_1}(x_1^{2003}), ..., \mu_{G_8}(x_8^{2003})] = 1.0 \]
\[ c.) \mu_{G_t}(X_{2003}) = \frac{1}{8}[\mu_{G_1}(x_1^{2003}) + ..., + \mu_{G_8}(x_8^{2003})] = 0.91 \]
\[ d.) \mu_{G_t}(X_{2003}) = \mu_{G_1}(x_1^{2003}) \cdots \mu_{G_8}(x_8^{2003}) = 0.45 \]
\[ e.) \mu_{G_t}(X_{2003}) = \omega_1 \mu_{G_1}(x_1^{2003}) + ... + \omega_8 \mu_{G_8}(x_8^{2003}) = 0.94 \]

Investments are divided into fractions according to the rule for division specified by the commune's authorities. This rule specifies what percentage of the investment as a whole is at the stage \( t \) assigned to \( i \)-th indicator. These expenditures do not add up to 100% as the commune authorities assign some amount as reserves and this amount has not been taken into account when developing the model.

### Table 1

<table>
<thead>
<tr>
<th>Quality of life expenditure expressed in per cents</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1^t ) transport and communication</td>
<td>2.92</td>
<td>5.13</td>
<td>10.92</td>
<td>5.65</td>
<td>10.99</td>
</tr>
<tr>
<td>( x_2^t ) municipal services and environmental protection</td>
<td>10.01</td>
<td>8.24</td>
<td>12.73</td>
<td>20.27</td>
<td>9.33</td>
</tr>
<tr>
<td>( x_3^t ) housing</td>
<td>8.34</td>
<td>1.83</td>
<td>1.41</td>
<td>9.06</td>
<td>9.816</td>
</tr>
<tr>
<td>( x_4^t ) education</td>
<td>38.88</td>
<td>47.60</td>
<td>40.79</td>
<td>20.18</td>
<td>36.83</td>
</tr>
<tr>
<td>( x_5^t ) culture and national heritage protection</td>
<td>3.94</td>
<td>4.86</td>
<td>3.44</td>
<td>26.33</td>
<td>4.145</td>
</tr>
</tbody>
</table>

Weight values for the individual quality of life indicators provided for in Table 2. are a quotient of the amount for investments of the given indicator (Table 1.) divided by the amount for investments of the total life indicator (\( \sum \) of all indicators at a given control stage).

### Table 2

<table>
<thead>
<tr>
<th>Quality of life indicators</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1^t ) transport and communication</td>
<td>0.03</td>
<td>0.05</td>
<td>0.11</td>
<td>0.06</td>
<td>0.11</td>
</tr>
<tr>
<td>( x_2^t ) municipal services and environmental protection</td>
<td>0.11</td>
<td>0.09</td>
<td>0.13</td>
<td>0.21</td>
<td>0.10</td>
</tr>
<tr>
<td>( x_3^t ) housing</td>
<td>0.09</td>
<td>0.02</td>
<td>0.01</td>
<td>0.09</td>
<td>0.10</td>
</tr>
<tr>
<td>( x_4^t ) education</td>
<td>0.41</td>
<td>0.49</td>
<td>0.43</td>
<td>0.20</td>
<td>0.38</td>
</tr>
<tr>
<td>( x_5^t ) culture and national heritage protection</td>
<td>0.08</td>
<td>0.05</td>
<td>0.04</td>
<td>0.27</td>
<td>0.04</td>
</tr>
<tr>
<td>( x_6^t ) physical culture and sports</td>
<td>0.03</td>
<td>0.03</td>
<td>0.02</td>
<td>0.01</td>
<td>0.04</td>
</tr>
</tbody>
</table>

### Table 3

<table>
<thead>
<tr>
<th>Quality of life expenditure expressed in per cents</th>
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<td>3.44</td>
<td>26.33</td>
<td>4.145</td>
</tr>
</tbody>
</table>

In the overall assessment of the model the following fuzzy constraint must be specified \( \mu_{c^{-1}}(u_{t-1}) \). Assuming it has a form of the function, two values must be specified with parts of the linear one. The first one specifies the amount of the totally acceptable investment, where \( \mu_{c^{-1}}(u_{t-1}) = 1 \), for \( u_{t-1} < u_{t-1}^p \). The second value presents the amount still to be accepted, below which \( \mu_{c^{-1}}(u_{t-1}) = 0 \), for \( u_{t-1} \geq u_{t-1}^p \). In the model under discussion, the amount of the commune’s budget for investments for a given control period is the fuzzy constraint. Moreover, for each quality indicator at the stage \( t = 1, ..., N \) an objective fuzzy sub-goal is being defined \( G_0^{i,t} \), described by the membership function \( \mu_G(x_i^t) \). \( G^{i,t} \) is fully attained where \( \mu_G(x_i^t) = 1 \), less preferred for \( 0 < \mu_G(x_i^t) < 1 \) and impossible by assumption, where \( \mu_G(x_i^t) = 0 \). In the example under analysis, the planned amount for investments from the approved budget is assumed as a desired...
value, while the real amount for investments is the value which is possible to accept.

For real data a value of the fuzzy decision was derived (according to 18), which will be used in assessment of the strategy for the region. The division of the investment for three cases was analysed. In the first one, it is assumed that investments will be divided equally at individual stages (years). In the second case, they will increase by the same value at each subsequent stage. In the third, real data from the Brzeg commune were provided for comparison.

<table>
<thead>
<tr>
<th>Case</th>
<th>Year</th>
<th>Amount</th>
<th>Amount</th>
<th>Amount</th>
<th>Amount</th>
<th>Amount</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0</td>
<td>4 200 000</td>
<td>7 200 000</td>
<td>3 600 000</td>
<td>5 400 000</td>
<td>6 400 000</td>
<td>1 200 000</td>
</tr>
<tr>
<td>II</td>
<td>0</td>
<td>4 200 000</td>
<td>7 200 000</td>
<td>3 600 000</td>
<td>5 400 000</td>
<td>6 400 000</td>
<td>1 200 000</td>
</tr>
<tr>
<td>III</td>
<td>0</td>
<td>4 200 000</td>
<td>7 200 000</td>
<td>3 600 000</td>
<td>5 400 000</td>
<td>6 400 000</td>
<td>1 200 000</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>4 200 000</td>
<td>7 200 000</td>
<td>3 600 000</td>
<td>5 400 000</td>
<td>6 400 000</td>
<td>1 200 000</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4 200 000</td>
<td>7 200 000</td>
<td>3 600 000</td>
<td>5 400 000</td>
<td>6 400 000</td>
<td>1 200 000</td>
</tr>
<tr>
<td></td>
<td>3</td>
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<td>7 200 000</td>
<td>3 600 000</td>
<td>5 400 000</td>
<td>6 400 000</td>
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</tr>
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<td>7 200 000</td>
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<td>6 400 000</td>
<td>1 200 000</td>
</tr>
</tbody>
</table>

Assessment of life quality according to the objective indicator assumes various values, depending on the accepted aggregating function. In the case of the minimum type of aggregation, the operator with the lowest value is accepted as the level of the total quality of life indicator. It can be concluded that low value of one indicator may not be compensated for by the high value of another one. Such an assessment is not correct, as social satisfaction stimulated by implementation of other indicators may considerably exceed the value than it is in the case of the minimum. The amount of the investment is also important. In some cases small or incomplete use of the big amount may prove to be better than 100% use of the smaller amount. The maximum type of aggregation may be interpreted similarly, where the highest operator indicates the degree of social satisfaction. The best fulfilled indicator is not always the most important for inhabitants. That is why the weighted sum or arithmetic mean is a good indicator as all components are taken into account.

The process of regional development is difficult to formalize, it is multidimensional and comprises many values which are not easy to specify. Values such as the process of attaining goals and sub-goals, establishing constraints for investments or selection of a suitable control strategy are blurred by nature. That is why multi-stage fuzzy control may prove to be the right model for analysis.

References

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