Behaviour of Two-Phase Machine Under Non-Harmonic Supply

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Abstract
This paper compares the single phase induction motor (SPIM) behavior under sinusoidal harmonic supply and no-sinusoidal supply by two phase inverter. The SPIM is supplied as a two phase induction motor (TPIM). The supply is simulated by Matlab Simulink by sinusoidal, rectangular and PWM supply. Measurement was done with sinusoidal and rectangular supply. The results are compared.

1. INTRODUCTION
Single-phase induction machines (SPIM) have been used for a long time because of their simple construction and because the single-phase power supply is available in almost each household. However, this advantage is compensated by minor efficiency of the machine and its complicated start-up.

Single-phase power supplied to single phase winding is a source of pulsating field in a machine that in analysis can be resolved into two equal revolving fields rotating in opposite direction. Thus, the machine does not produce starting torque (Fig. 1).

![Fig. 1 Torque-speed characteristic for SPIM induced positive and negative sequence components](image)

After the rotor starts rotating in one direction by a mechanical impulse, the field rotating in direction of the rotation of rotor starts to dominate over backward rotating field and motor will run up. Torque of backward rotating field will decrease with the increasing rotor speed and at nominal speed it is only a very small fraction of total torque.

As can be seen in the Fig. 1 there is no starting torque which is the biggest disadvantage of SPIM. Thus, SPIM with only one stator winding (see Fig. 2a) are not manufactured. Usually an auxiliary winding with different impedance shifted in space by 90° is wound to ensure the run-up and improve the starting up performance of SPIM (Fig. 2b).

![Fig. 2 Possible connections of single-phase IM: a) supplied from main phase b) with capacitor c) two-phase supply](image)

There are various methods for running-up the SPIM with auxiliary winding. The most commonly used method is the one with permanently connected capacitor (Fig. 2b). With respect to the current of the main winding the current in auxiliary winding is now shifted by almost 90° and leading the main winding current. The field produced by both windings is no more pulsating and the machine produces starting torque. However, field produced in machine air gap is not symmetrical like in three-phase machines but an elliptical one which means that the magnitude of magnetic field density along the machine air gap is non-uniform (Fig. 3). Such field is a source of increased noise and vibrations due to radial forces affecting the stator boring [1].
Permanent progress in the field of power electronic devices has given a rise to two-phase induction machines. These machines have two equal stator windings spatially shifted by 90° (Fig. 2c). Supply is carried out by two-phase power converter with voltages shifted by 90° in time. By means of this supply a waveform of flux density rotating in the air gap, similar to that of the field in three-phase machines, is produced and vibrations and unfavourable noise are thus suppressed. Due to smaller amount of power switches in converters for TPIM and their simpler construction in comparison with three-phase converter, such machines are quite economical. Of course, their torque and output power are proportional to number of phases.

An equivalent circuit of SPIM in steady-state condition can be created if rotor parameters are equally divided between forward and backward components of rotating field. Forward resistance component is represented as $0.5 R_r' / s$, backward component as $0.5 R_r'/ (2 - s)$, which can be seen in Fig. 4 and justified by the Fig. 1. Magnetizing reactance $X_\mu$ is divided into two components as well.

\[ Z_{rp} = 0.5 X_r' + \frac{0.5 R_r'}{s} \]  
\[ Z_m = 0.5 X_r' + \frac{0.5 R_r'}{2 - s} \]

where $R_r'$ and $X_r'$ are rotor resistance and reactance referred to the stator side.

Together with magnetizing reactance $X_\mu$ it results in total parallel impedances:

\[ Z_r = \frac{Z_r' \cdot 0.5 X_r'}{Z_{rp} + 0.5 X_r'} \]  
\[ Z_s = \frac{Z_s' \cdot 0.5 X_s'}{Z_{rn} + 0.5 X_s'} \]

After adding the stator impedance $Z_s = R_s + j X_s$ we obtain the input impedance $Z_{in}$. Afterwards, input current $I$ can be calculated as ratio of input voltage and input impedance. Positive and negative sequence of rotor current can be calculated on the basis of equal voltage $E_p$: $Z_{rp} I_p = Z_m I_m$, and $E_o$ respectively.

\[ I_{rp} = \frac{Z_{rp} I_s}{Z_{rp}} \]  
\[ I_m = \frac{Z_m I_s}{Z_m} \]

Torque is defined as difference between its components for forward and backward rotating fields (Fig. 1)

\[ T = \frac{p}{\omega} \left( I_{rp}^2 \frac{0.5 R_r'}{s} - I_m^2 \frac{0.5 R_r'}{2 - s} \right) \]

From eq. (1)-(7) can be seen that parameters $R', X', X_\mu$, $R_s$, $X_s$ are needed to be able to simulate torque in steady-state condition as well as in transients (see chap. IV.).

The following conditions by two-phase supply are to be followed: magneto-motive force (MMF), that means the winding current multiplied by number of effective turns, has to be the same in both phases and the voltages are to be shifted by 90 degrees. If the above mentioned conditions are kept, the backward field of main winding is entirely eliminated (see Eq. 8). The phasor sum of the MMF waves generated by the main and auxiliary (subscripts $m$ and $a$) windings will be given by:

From equivalent circuit the rotor impedance belonging to forward (subscript “p” as positive) and backward (subscript “n” as negative) rotating fields can be calculated as:
The parameters needed for the simulation are [7]:

\[
F_{2a} = F_m + F_a = A_m [0.5 \sin(\alpha - \theta) + 0.5 \sin(\alpha + \theta)] + A_a [0.5 \sin(\alpha - \pi/2 + \theta + \pi/2)] + A_a [0.5 \sin(\alpha - \pi/2 + \theta - \pi/2)]
\]

The total MMF generated by symmetrical two-phase supply will be:

\[
F_{2m} = A \sin(\alpha - \theta)
\]

where \(\theta\) is mechanical angular position in air-gap, \(F_m\) is MMF generated by the main winding, \(F_a\) is MMF generated by auxiliary winding, \(A_m\) and \(A_a\) are current dependent constants of MMF waves of the main and auxiliary winding respectively. By symmetrical condition \(A_m = A_a = A\).

In the next, real TPIM is investigated with the following name plate:

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>NAMEPLATE OF INVESTIGATED MACHINE</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_n) [W]</td>
<td>(V_n) [V]</td>
</tr>
<tr>
<td>150</td>
<td>230</td>
</tr>
</tbody>
</table>

The parameters needed for the simulation are [7]:

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>PARAMETERS OF THE MACHINE</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAIN WINDING</td>
<td>(R_r) [Ω]</td>
</tr>
<tr>
<td></td>
<td>19.92</td>
</tr>
<tr>
<td></td>
<td>21.37</td>
</tr>
<tr>
<td>AUXILIARY WINDING</td>
<td>(R_a) [Ω]</td>
</tr>
<tr>
<td></td>
<td>21.32</td>
</tr>
<tr>
<td></td>
<td>22.3</td>
</tr>
<tr>
<td></td>
<td>(L_m) [H]</td>
</tr>
<tr>
<td></td>
<td>1.26</td>
</tr>
</tbody>
</table>

2. SIMULATION MODEL

For purposes of analysis, the TPIM can be considered to be an unsymmetrical two-phase machine. The analysis of such machines can be conveniently handled by a \(d-q\) model approach used for three-phase machines. Details of the derivation of the \(d-q\) model are described in [6]. If all \(q\) quantities are referred to the main winding and all \(d\) quantities are referred to the auxiliary winding and all rotor quantities are referred to stator side the voltage equations can be expressed as:

\[
v_{qr} = \frac{d\psi_{qr}}{dt} + R_{qr}i_{qr}
\]

Flux linkages can be written as:

\[
\psi_{qr} = L_{qm}i_{qr} + L_{qm}(i_{ds} + i_{qr})
\]

An expression for the instantaneous electromagnetic torque can be obtained by applying the principle of virtual displacement. This relation (positive for motor action) is expressed as:

\[
T_e = p \left( \frac{N_a}{N_m} \psi_{qr}i_{ds} - \frac{N_a}{N_m} \psi_{dr}i_{qr} \right)
\]

where \(p\) is a number of pole-pairs. The simulated outputs are seen on the Fig. 9, 10, 11, 13, 14.

3. EXPERIMENTAL RESULTS

To verify the simulated outputs, the behavior of the machine under various load conditions was investigated by a mechanical coupling of the
investigated machine to permanent magnet DC machine via torque transducer MBM T20WN (Fig. 5).

Consequently, a comparison between simulated and measured torque-speed characteristic was done.

3.1 TPIM supplied by sinusoidal voltage

The TPIM is supplied by two-phase sinusoidal voltage. There is used connection of two one-phase transformers with adjustable capacitor instead of converter (Fig.6). The capacitor is used to generate the 90 electrical degrees displacement between phases. By this method the currents have equal amplitude and they are shifted 90 electrical degrees. Then it is possible consider rotating field in TPIM (see Eq.8). This connection provides best results, because of harmonic supply. The backward field is fully suppressed and magnetic field in machine air-gap is symmetrical.

3.2 TPIM supplied by converter

It is possible to create two-phase supply by the converter (Fig.7). The converter causes a lower performance in comparison with sinusoidal supply, because of high harmonic components of converter supply. The output power is provided only from fundamental harmonic component.

In our case is used converter with rectangular shape of phase voltage (Fig.9). This voltage waveform has very high account of harmonics.

The measured and simulated torques of TPIM are compared, see Fig. 10.

4 RESULTS AND THEIR COMPARISON

The torque versus rotational speed was measured by sinusoidal and rectangular voltage. These real values are compared with simulation by sinusoidal, rectangular and switched PWM voltage at which switching frequency was 1 kHz. All obtained results are in one graph for better comparison (Fig.10). Simulated rectangular supply has good coincidence with measured rectangular supply. It is confirmed, that low switching frequency (rectangular supply has 100 Hz) has poor effect on the motor. With higher frequency (1kHz) closer to sinusoidal waveform is the output torque higher, close to sinusoidal supply. Simulated torque waveforms are higher than measured waveforms, because the simulated model does not take in the account the losses of TPIM. This error is about 11%. In the Fig.10 it is seen, that the rectangular and PWM supply decreases starting torque...
about 28.3% and 4.5%, respectively, in comparison with sinusoidal supply. Around rated speed the rectangular and PWM supply decreases developed torque about 15% and 7% respectively in comparison with sinusoidal supply. PWM supply provides higher torque then rectangular, but lower torque then sinusoidal supply.

![Graph showing comparison of simulated and measured waveforms]

**Fig.10 The comparison of simulated and measured waveforms: 1- simulation by rectangular voltage, 2- measurement by rectangular voltage, 3- simulation by sinusoidal voltage, 4- measurement by sinusoidal voltage, 5- simulation by PWM 1 kHz**

5 CONCLUSION

The simulated torque characteristic was compared with real measurement and the results give good coincidence. It is seen (in Fig.10) that the non-harmonic rectangular voltage supply decreases the TPIM performance, because of high harmonic spectrum. By using of PWM it is possible to reach almost the performance as by sinusoidal supply. The PWM supply was only simulated in Matlab-Simulink. Next work will be carried out real supply with PWM inverter and investigation of switching frequency on TPIM losses.

6 REFERENCES


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