ENERGY METHOD OF NONLINEAR INDUCTANCE PARAMETERS IDENTIFICATION

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Abstract

The energy method is used in the analysis of complex energy modes of electrical circuits and systems. It can be used for identifying particular energy modes and the nature of the exchange and storage processes between supply and individual elements of the consumer, as well as to perform the identification of nonlinearity parameters based on the analysis of real signals in the energy conversion circuits. The potential of the energy method consists in the solution of the problem of the nonlinear inductance parameter identification. This approach is based on the balance equations of power elements of the source and the consumer on each harmonic and allows obtaining the required number of equations for determining the parameters. A new method for the determination of the constant component of the nonlinear inductance is suggested.

Introduction

Nowadays one of important directions in electrical mechanics consists in the search of ways allowing one to accurately describe processes caused by the presence of nonlinear elements. The analysis showed that under the analysis of physical processes in electrical mechanics the power system elements balance equation are often used in the form of simple dependences which characterize the integral values of active power and power losses. Balance of reactive and active power for above mentioned elements is used rarely.

Theoretical studies and the developments of different authors related to the problems of the instantaneous power has shown the inadmissibility of the usage the concept of apparent power in problems of energy modes estimation, especially with complex voltage and current harmonic waves [1].

It should be mentioned that the processes that are characterized by the energy exchange between the source and the consumer, are complicated, and its singularities in the mathematical description for electrical circuits, and for the technological mechanisms of the various purposes [2, 3].

The development of the energy method using the apparatus of the instantaneous power has significantly expanded the boundaries of effective usage of the results analysis to estimate the parameters of equivalent circuits of the objects, determine the quality of energy transformation, estimation the influence of energy conversion processes on the electromechanical systems and complexes behavior [4].

It is obviously that usage of the energy method in the problems of diagnostics and parameters estimation of electrical systems will improve the quality of the energy processes analysis. This approach gives mathematical basis for further analysis of the energy conversion process between any part of the consumer and the source. Thus, the investigation of the possibility of using energy method both for energy processes analysis and the nonlinear circuits parameters identification is topical.

Problem statement

In modern engineering, electro-mechanical systems are widely and diversely used. These systems consist of motors, generators, converting devices, etc. The different properties and characteristics of each circuit element must be taking into account for describing the dynamic and energetic processes in such systems. An electromechanical systems and complexes with non-linear elements are most difficult for analysis and estimation of energy processes. Difficulties while calculation the circuits with non-linear elements consists in determining varying characteristics depended on the circuits’ parameters, load, time, etc.

An importance of the solution the problems on the study of nonlinear circuits caused by the necessity of taking into account the physical phenomena which influence on the energy conversion processes in the circuit.

Usage the energy method for analysis the energy exchange processes allows one get a more informative assessment because of taking into account the real physical processes in the circuit.
The analysis shows the efficiency of the energy method for the identification and analysis of the energy modes of linear and nonlinear circuits. The method was developed in the early 2000s after a series of papers on the subject, published in Ukraine [5, 6].

Materials of Research

The essence of the method is the following: assume that the structure of the circuit, including power supply and a consumer with a nonlinear load, is given. For simplicity, the consumer can be considered as a set of active resistance and non-linearity (Fig.1).

![Fig.1. Equivalent circuit of an electric circuit with nonlinear inductance](image)

Analysis was carried out, assuming that the non-linearity is the inductance with saturation. Non-linearity implies that the load current is described by following dependence:

\[ I(t) = \sum_{m=0}^{M} I_m \cos (m \omega t + \phi_m), \]

where \( I_m \) is the peak value of current to the m-th harmonic, \( I_m \) is the cosine component of the current, \( I_{mb} \) is the sine component of the current, \( \Omega_m \) is the angular frequency of the m-th harmonics, \( \phi_m \) is the sine angle between voltage and current, \( M \) is the harmonic number.

If \( U(t) \) is harmonic waveform \( (U(t) = \cos(\omega t)) \), the instantaneous power source in this context is the product of voltage and current:

\[ P(t) = U(t)I(t) = \sum_{k=0}^{K} P_{kw} \cos(k \omega t) + \sum_{k=0}^{K} P_{kw} \sin(k \omega t), \]

where \( P_{kw} \) is the constant component of the total instantaneous power; \( P_{kw} \) is the cosine component of the instantaneous power of the canonical order; \( P_{kw} \) is the sine component of the instantaneous power of the canonical order; \( P_{kw} \) is the sine component of the instantaneous power of non-canonical order; \( k \) is the number of harmonic power.

According to the results, the instantaneous power of the source in any moment is equal to the sum of the instantaneous power components of the individual components:

\[ P_{kw}(t) = P_{kw}(t) + P_{kw}(t). \]

where \( P_{kw}, P_{kw}, P_{kw} \) are the power on the power supply, active resistance and inductance, respectively.

According to the Telledzhen's theorem and research [7], the power equality of the power source and the consumer meets the energy conservation law for a single harmonic and for the whole number of harmonics, i.e. for all instantaneous power harmonics. This law is fundamental in assessing the unbiased result.

To obtain the general dependence of the energy balance equation, let represent the power source in the following form:

\[ P_{kW}(t) = \sum_{k=0}^{K} P_{kw} \cos(k \omega t) + \sum_{k=0}^{K} P_{kw} \sin(k \omega t), \]

where \( U_{kW}(t) = L \frac{dI(t)}{dt} \).

Thus, the energy conservation law holds for the circuit “the source - the consumer” on each of the harmonics. It should be mentioned that signals of instantaneous power include both constant and alternating components. If the harmonic structure of the voltage and current is known, it is possible to construct a power balance equations system on each of the harmonics:

- for constant component;
- for the cosine components;
- for the sine components.

Moreover, the number of equations depends on the current and voltage harmonics number. For example, if the number of current harmonic is equal to M, and the number of voltage harmonics is equal to N, then under condition of \( M = N \), we have M equations with a constant component, M with the cosine and sine components. Taking into account the fact that the number of harmonics of the active component is added \( (\sum_{k=0}^{K} P_{kw}) \), we obtain a system of \( 2M + 1 \) equations, where the power supply components are on the left side, and consumer elements are on the right. If these elements are unknown, they can be easily identified by solving a system of equations. The basics of usage the energy method for solving such problems are represented in paper [8].

When the energy method is considering for non-linear elements (in this case for saturated inductance) the possibility for taking into account specific features of such circuits, because of non-linear ele-
ments, appears. One from such feature is the appearance of higher harmonics, even if the circuit with a sinusoidal voltage source is under analysis.

Dependence $\psi(I)$ is the main characteristic of the nonlinear inductance. Nonlinear properties determined by presence of a ferromagnetic core, for which the dependence of the magnetic induction on the field strength is nonlinear. Excluding the effects of magnetic hysteresis, the inductance is characterized by a static (constant) component $L_0 = \frac{\psi}{I}$ and a differential (variable) component $L(I) = \frac{d\psi(I)}{dt}$. If we consider that inductance is supplying with direct and alternating current, in the first case, the magnetic flux is constant, while in the second — the variable. In this case, the dependence of $L(I)$ and $L_0$ will be changed with the current changes and the task of their definition became more complicated. It should be noted that the parameter $L_0$ that was previously presented as a constant, in this case will be a function, depending on the saturation parameters of $L(I)$, i.e. depending on the current in the circuit, or that the same thing — from the supply voltage. This conclusion explained by the features of the formation of the instantaneous power, which allows to take into account all the components of energy conversion processes. Thus, definition of the dependence $L_0 = f(I)$ contains a point of interest for determining the parameters of the nonlinear inductance.

The analysis showed that the solution of the above mentioned problem was given in sufficient attention in the absence of mathematical formalism which allows one to take into account features of the physical processes occurring in the nonlinear inductance. A way for determination the parameters of nonlinear energy method based on the instantaneous power, is described following.

As an example, let consider an electric circuit where the nonlinear element is taken from the saturation of the inductance $L(t)$ (Fig. 2).

![Fig.2. The dependence inductance of the current](image)

Variation of the inductance of the current is described by the relation:

$$L(I) = a_0 + a_2I^2 + a_4I^4,$$

(2)

where $a_0$, $a_2$, $a_4$ are approximation coefficients, which represent complex relationships, resulting from the transformation frequency of the trigonometric function that describes the current.

In order to obtain the dependence of $L(t)$ we must substitute the values of $I(t)$ in $L(t)$. The coefficients $a_0$, $a_2$, $a_4$ are unknown. The dependence of $L(t)$ is a set of trigonometric functions, which, when they are raised to an even degree, gives constant and alternating components. Thus, the expression for $L(t)$ becomes following:

$$L(t) = a_0 + a_2I^2(t) + a_4I^4(t).$$

(3)

Let consider the above described definition of the coefficients in the example. In our case, a typical symmetric magnetization curve is taken for the analysis, which is characterized by the change of inductance with double frequency compared to the frequency components of the current process of determining the magnetization of the material.

Taking into account given parameters of the initial magnetization curve, we can get the current parameters experimentally or by simulation. Using the description of nonlinearity by means of defined differential equations, we can express the dependence current from time [9]. After mathematical transformation, the curve will be represented by a trigonometric series, which includes the 1, 3, 5 current harmonics:

$$I(t) = I_{1a} \cos(\Omega t) + I_{3a} \sin(3\Omega t) + I_{5a} \cos(5\Omega t) + I_{5a} \sin(5\Omega t);$$

(4)

Then, the expression for the nonlinear inductance becomes the following:

$$L(t) = L_0 + a_2 \left( \sum_{m=1}^{\infty} (I_{ma} \cos(m\Omega t) + I_{ma} \sin(m\Omega t)) \right)^2 + a_4 \left( \sum_{m=1}^{\infty} (I_{ma} \cos(m\Omega t) + I_{ma} \sin(m\Omega t)) \right)^4,$$

where $L_0 = a_0 + f(a_2; a_4; I_1; I_3; \ldots; I_m)$.

Analysis of the obtained dependence of inductance shows that the constant component $L_0$ is a function that includes the amount of inductance saturation parameters: a constant $-a_0$ and the function of the variable components, obtained by transformation of trigonometric series current, which is known to lead to the emergence of the permanent components. Literature analysis [10, 11] has shown that studies of this kind have not previously performed and such an approach, by definition $L_0$, is considered the first time. The main feature in determining the parameters of the inductance saturation curve by energy method is the possibility for obtaining the curve $L(t)$ in terms of parameters $a_0$, $a_2$, $a_4$, which greatly simplifies the calculation and makes it possible to obtain simultaneously and the parameters of the nonlinearity and saturation. In addition to the above described unknown parameters any EMS parameters can be determined, which will be shown later in this paper.

Let compose the power balance equation between the source and the consumer for determina-
tion the above described parameters. Power on each element:

output power supply

\[ P_{l}(t) = U_{l}(t) I_{l}(t) = (U_{l1} \cos(\Omega t) + U_{l2} \sin(\Omega t)) \times \]

\[ (I_{l1} \cos(\Omega t) + I_{l2} \sin(\Omega t) + I_{l3} \cos(3\Omega t) + I_{l4} \sin(3\Omega t) + I_{l5} \cos(5\Omega t) + I_{l6} \sin(5\Omega t)) \]  \( \text{(6)} \)

the active resistance

\[ P_{R}(t) = U_{R}(t) I_{R}(t) = R(I_{R1} \cos(\Omega t) + + I_{R2} \sin(\Omega t) + I_{R3} \cos(3\Omega t) + I_{R4} \sin(3\Omega t) + + I_{R5} \cos(5\Omega t) + I_{R6} \sin(5\Omega t)) \]  \( \text{(7)} \)

the nonlinear inductance

\[ P_{L}(t) = U_{L}(t) I_{L}(t), \]

\[ U_{L}(t) = (L_{0} + L_{1} \frac{dI(t)}{dt} + + L_{2} \frac{d^{2}I(t)}{dt^{2}}) \]  \( \text{(8)} \)

where

In this non-linear inductance is:

\[ L(t) = L_{0} + a_{1} \left( \sum_{m=1}^{M} \sin(m\Omega t) \right)^{2} \]

\[ + a_{2} \left( \sum_{m=1}^{M} \cos(m\Omega t) \right)^{4} \]  \( \text{(9)} \)

where \( L_{0} = a_{0} + a_{2} \left( \sum_{m=1}^{M} (I_{m})^{2} \right)^{2} \) + + \( a_{4} \left( \sum_{m=1}^{M} (I_{m})^{4} \right)^{4} \).

Let the current and voltage are known, and resistance R and the parameters of the saturation inductance \( L/I = a_{0}, a_{2}, a_{4} \) are unknown.

For composition the equations we use the expression balance of instantaneous power (1) on the elements of the circuit.

Taking into account that the nonlinear inductance of the unknown parameters is expressed in terms of saturation, this approach allows us to define not only the parameters of saturation, but also the nonlinearity parameters, which will be shown later.

According to the superposition principle, system identification equations in general form are the following:

\[ U_{1a} I_{1a} + U_{2b} I_{2b} = R_{1a} I_{1a} + R_{2b} I_{2b} + R_{1c} I_{1c} + R_{2d} I_{2d} + + R_{3a} + a_{1} \left( \sum_{m=1}^{M} (I_{m})^{2} \right) + a_{2} \left( \sum_{m=1}^{M} (I_{m})^{4} \right) \]  \( \text{(10)} \)

\[ U_{1a} I_{1a} + U_{1a} I_{2b} - U_{2b} I_{1a} = R_{1a} I_{1a} - R_{2b} I_{2b} + + 2R_{1b} I_{1a} + 2R_{2b} I_{2b} - 2R_{1b} I_{2b} + + 2R_{2b} I_{1a} + 2R_{1b} I_{2b} + 2R_{1a} I_{5a} + + a_{0} \left( \sum_{m=2}^{M} \cos(2\Omega t) \right) + + a_{1} \left( \sum_{m=2}^{M} \cos(2\Omega t)^{2} \right) \]  \( \text{(10)} \)

\[ U_{1a} I_{1a} + U_{1a} I_{2b} + U_{1a} I_{3a} - U_{2b} I_{1a} = 2R_{1a} I_{1a} + + 2R_{1b} I_{2b} + 2R_{3a} I_{3a} + + 2R_{1a} I_{3a} + 2R_{1b} I_{2b} + 2R_{1a} I_{3a} + + a_{0} \left( \sum_{m=2}^{M} \sin(2\Omega t) \right) + + a_{1} \left( \sum_{m=2}^{M} \sin(2\Omega t)^{2} \right) \]  \( \text{(10)} \)

\[ -U_{2b} I_{1a} + U_{1a} I_{3a} + U_{2b} I_{3a} + U_{2b} I_{3a} = 2R_{1b} I_{1a} + + 2R_{1a} I_{3a} + 2R_{1b} I_{2b} + 2R_{1a} I_{5a} + a_{0} \left( \sum_{m=4}^{M} \cos(4\Omega t) \right) \]

\[ + a_{1} \left( \sum_{m=4}^{M} \cos(4\Omega t)^{2} \right) \]  \( \text{(10)} \)

Analysis of the equations showed that the number of equations exceeds the number of identification parameters of equivalent circuit, which is explained by the formation of components of the instantaneous power by harmonic analysis of the product source voltage and current signals [12].

A convenient tool for solving such systems is a direct method for solving linear equations using Gauss determinant [9].

The study was performed for a circuit where for the inductance of the current curve (Fig. 2) the power supply voltage is \( U(t) = 220 \cos(\Omega t) \); angular frequency is \( \Omega = 100c^{-1} \); current in the circuit consists of 1, 3, 5 harmonic (Fig. 4):

\[ I(t) = 1.37 \cos(\Omega t) - 1.01 \sin(\Omega t) + 0.08 \cos(3\Omega t) - 0.3 \sin(3\Omega t) + 0.017 \sin(5\Omega t) - 0.085 \sin(5\Omega t). \]  \( \text{(10)} \)

![Fig.3. Voltage and current curves](image)

Taking into account mentioned parameters, the system of equations will be the following:

\[ 150 = 3.1a_{2} - 4.8a_{4} + 1.5R; \]

\[ 160 = -3385a_{4} - 171.5a_{0} - 875a_{2} + 0.9R; \]

\[ -144 = -2273a_{4} - 870a_{0} - 555a_{2} - 1.7R; \]

\[ 10.6 = 1078a_{2} - 5111a_{4} - 117a_{0} - 0.09R; \]

As the solution, the following parameters were defined: \( R = 99 \text{ Ohm} \); \( a_{0} = 1.44; a_{2} = -0.4; a_{4} = 0.05 \).

Thus, the expression for \( L/I \) becomes the following:

\[ L/I = 1.44 - 0.41 + 0.05t \]  \( \text{(11)} \)

The parameters \( L/I \) can be taken from substitution the values of \( I(t) \) in (11).

We consider separately the receipt of dependency \( L_{0} = f(I) \). As it was shown above, the parameter \( L_{0} \) value is not constant and has a complex mechanism of the formation, depending on the current change.

Preparation of the curve \( L_{0}(I) \) by changing the current or as mentioned above-stress. In this case, we would consider an example of changing the voltage.

The mechanism for definition \( L_{0} \), was described above. In order to get different values of the corresponding change in the voltage, it is necessary to write the system of equations of power balance at each harmonic using the expression for the compilation of the equations (1). Since the coefficients of the approximation of the curve \( L/I \) will be constant (the curve does not change), the parameter \( L_{0} \) depends on
determine the parameters of nonlinear circuits. It was shown that the parameter $L_0$ value is also unstable and has a complex dependence on changes in current and voltage, respectively.

**Conclusions**

It is shown that the energy method is effectively applied for solving practical problems of identification of electrical circuits while nonlinearity is present. The main advantages of this method are:
- the possibility of solving the problem of determining the parameters of the nonlinear saturation of the inductor and other elements of nonlinear characteristics and properties;
- energy method is universal and applicable to various kinds of systems that incorporate elements of the complex nonlinear dynamic characteristics;
- this method is based on the law of energy conservation;
- the application of the instantaneous power method for the system of equations for each harmonic allows to obtain the necessary number of equations to determine the identity of the parameters of electrical circuits.

In the future we plan to define the parameters of the nonlinear inductance taking into account losses in the steel.

**Bibliography**


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