Calculation of capacitance of two spiral coils made on printed circuit board

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**Abstract**

The aim of this paper is to calculate the self-capacitance of double-layer spiral coil built on printed circuit board. The layers of coil are placed opposite to each other and the material of board serves as dielectric material. First, the capacitance of such coil is determined using analytic approach. Then the capacitance is calculated using finite elements method of solution. For this purpose the software package called COMSOL Multiphysics – AC/DC module is used. Designed system is modeled in 2D axisymmetric space dimension as electrostatic problem, results are compared with analytic solution and difference between the solutions is obtained.

1. **Introduction**

COMSOL Multiphysics is software package that solves systems of coupled three-dimensional partial differential equations. The COMSOL Multiphysics engineering simulation software environment facilitates all steps in the modeling process – defining geometry, meshing, specifying physics, solving, and then visualizing results.

Model set-up is quick, thanks to a number of predefined physics interfaces for applications ranging from fluid flow and heat transfer to structural mechanics and electromagnetic analyses. Material properties, source terms and boundary conditions can all be arbitrary functions of the dependent variables. This software can be used to model different physical phenomena including electromagnetics. It uses the finite element method of solution and can model the complex problems, geometries or material properties in 2D or 3D representation for better understanding some phenomena.

In this paper COMSOL Multiphysics solves electric fields between two spirals placed (double – layer coil) opposite to each other made on printed circuit board and calculates the capacitance of such geometry. For calculation of self-capacitance of coils there are several analytic approaches and models. Due to the complex nature of the approximations, where the coil’s geometry has to be considered, the analytic approach can be overly difficult. High accuracy and precision during design stage is essential, because when printed on circuit board, the inductor can’t be changed anymore. Thus the using of finite element method can be a good alternative of solving such problems and can give us information about parameters of the coil in some range and can provide us with some initial conditions in designing such devices. But in our case the analytic approach is quite simple because for calculation is used only one simple formula. But the accuracy of such approach is very questionable.

2. **Problem formulation**

The spiral coil is a spiral – shaped formation made of copper. This copper spiral is electrically conductive showing an electrical resistance for the direct current. For the alternating current the inductor shows, except the electrical resistance, also a property called inductance, which describes the capability of producing a magnetic field around the inductor. From the electrical point of view the inductance acts as complex impedance being proportional to the frequency of alternating current. The ideal inductor shows only the inductance while its electrical resistance is negligible [1].

3. **The coil geometry**

The proposed geometry of such design is shown in Fig.1.

![Fig.1. Designed planar spiral coil – top view](image)
The geometry is spiral-shaped formation made of copper. This copper spiral is electrically conductive showing an electrical resistance for the direct current. Proposed one layer is physically described by the following parameters: \( d_{\text{in}} \) as the inner diameter [mm], \( d_{\text{out}} \) is outer diameter [mm], \( s \) expresses the spacing between turns [mm], \( w \) is the width of conductor [mm], and \( h \) is height of conductor [mm]. The second spiral is exactly the same and distance between the spiral is \( d=1,5 \) mm, which is the thickness of pertinax, material used in printed circuit boards. In this case this material is used as a dielectric material. The side view of coil made on PCB is depicted in Fig.2.

![Fig.2. Designed planar spiral coil – side view](image)

4. The coil capacitance

4.1 Analytic Solution

Capacitance of such geometry can be calculated by following equation:

\[
C = \varepsilon_r \varepsilon_0 \frac{S}{d}
\]

(1)

Where \( \varepsilon_0 \) is the permittivity of vacuum, \( \varepsilon_r \) is of dielectric between plates, \( S \) is the area of spirals and \( d \) is distance between both spirals. The area of spiral is calculated by neglecting the spacing between turns.

4.2 Model of designed geometry

The model (Fig. 3) is created in program COMSOL Multiphysics in AC/DC module as an electrostatics problem. The problem is solved in 2D axisymmetric space dimension. System is surrounded with an air domain and its boundary serves as electric insulation.

But in this case the geometry used in model is not exact spiral geometry. For solving such problem in 2D axisymmetric space dimension the geometry must be adapted, because spiral shaped formation is not axisymmetric. One layer of such adapted model is shown in Fig 4. In Fig. 5 3D representation of such coil is shown. Designed board is square shaped, but again, our model is solved in 2D axisymmetric space dimension and in such dimension we cannot use square shape.

![Fig.3. Model used in simulation](image)

![Fig.4. Adapted model used in simulation](image)

![Fig.5. 3D representation of model](image)

But circle shaped board instead. In both cases of adaptation; spirals and board, the solution accuracy (considering the length of copper) will be influenced in minimum way, because the length of copper path in designed and adapted spiral is the same and the board is larger than spiral in both cases.
5. Results

5.1 Analytic Solution

The capacitance of such geometry using (1) is $296 \times 10^{-10}$ F.

5.2 Simulation results

For simulation the model shown in Fig.3 (created from design in Fig.2) was used. As the material of spiral was selected copper with conductivity $\sigma = 5.997 \times 10^7$ S.m$^{-1}$. As terminal, according to Fig.2, upper spiral was selected and as a ground, lower spiral as shown in Fig.6. The voltage of terminal was set to 1V. Relative permittivity of pertinax is $\varepsilon_r = 5$.

Fig.6. Setting the terminal and ground

The parameters used in calculation in simulation are:

- Number of turns: $N = 9$
- Outer diameter: $d_{out} = 142$mm
- Inner diameter: $d_i = 30$mm
- Width of conductor: $w = 5$mm
- Spacing between turns: $s = 1$mm
- Height of conductor: $b = 0.8$mm
- Distance of coils: $d = 1.5$mm
- Area of board: 150x150mm

Very important parameter of simulation is a mesh. Proper size and shape of mesh elements influences the accuracy of solution; the finer the mesh elements, the better the solution accuracy.

Of course, very fine mesh can be a problem and computer cannot solve such model. It is very important to find optimal number of elements. In our case the triangular mesh of 293466 elements with maximum size of 2mm was created.

Capacitance of model is then determined from total electric energy using the following formula.

$$W_e = \frac{1}{2} CU^2$$

The problem was solved as electrostatic problem and calculation took 18s. Total electric energy is $W_e = 2.0694 \times 10^{-10}$ J. In the Fig.7 is shown electric potential and in the Fig.8 electric field is depicted. The capacitance using (2) is $C = 414 \times 10^{-10}$ F.

6. Conclusion

The aim of the paper was to estimate capacitance of two spirals made on printed circuit board using finite elements method of solution and comparing the results with analytic solution. First, in analytic approach we calculate the capacitance using formula, then using the software package we can calculate the capacitance directly.

The software solved the total electric energy and using simple formula for calculation electric energy we can calculate the capacitance. The difference between results of analytic and simulation solution is 28.5%. The finite element method of solution can be used for estimating other important properties of coil such as: inductance or resistivity of the coil and offers good solution accuracy. The future work will be focused on these properties and comparing them with experimental results.
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Bibliography


