Bit and Power Allocation Strategies in SVD-equalized Broadband MIMO Systems

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Abstract

Adaptive bit and power loading schemes in broadband Multiple-Input Multiple-Output (MIMO) systems are generally aiming to maximize the overall system throughput. However, in delay-critical applications such as voice or video streaming, a fixed data rate is crucial. Therefore in this contribution, bit and power allocation schemes in broadband MIMO systems for fixed rate applications are developed. The proposed schemes aim for optimizing the number of active MIMO layers and the number of bits per symbol in addition to an appropriate power assignment. The main goal is to minimize the overall bit-error rate (BER) for a given fixed data rate and limited transmit power. It turned out that it is not necessary to activate all MIMO layers in order to optimize the BER performance. Moreover, it can be seen from computer simulation results that multipath propagation is not a limiting factor in broadband MIMO systems.

1. Introduction

By exploiting the spatial dimension with systems composed of multiple transmit and receive antennas, also called MIMO systems, the spectral efficiency can be widely improved compared to the conventional Single-Input Single-Output (SISO) systems [1, 2]. Therefore, MIMO communication architecture has become crucial in future high-rate wireless applications. In systems with perfect channel state information (PCSI), further improvements to the system throughput can be achieved by adapting the system parameters to the varying channel state information, i.e., implementing adaptive modulation (AM) technique [3]. Combining MIMO system architecture with AM techniques, the performance of the system can be enhanced considerably [4, 5]. However, in real-time interactive applications, a fixed data rate is more preferable in comparison with system throughput improvements. Therefore, it is necessarily demanded to develop adaptive bit and power allocation schemes which optimize the BER performance for a given fixed data rate. Bit auctioning and power assignment strategies have attracted high attention in frequency non-selective MIMO systems and reached a state of maturity [6]. By contrast, bit and power allocation schemes in broadband MIMO systems require further substantial research.

In order to minimize the overall MIMO system BER, additional degrees of freedom are introduced by employing adaptive resource allocation schemes. However, the system-inherited interference such as inter-antenna interference and inter-symbol interference (ISI), introduced by the frequency selective channel, tends to decrease the overall system BER performance. Thus it requires appropriate handling. A popular signal processing strategy based on Singular Value Decomposition (SVD) has been widely used to resolve the overall interference in broadband MIMO systems [6, 7, 8]. Regarding this background, the novel contribution of this paper is to investigate the efficiency of implementing bit and power allocation schemes for SVD-equalized broadband MIMO systems with PCSI. The proposed adaptive schemes aim for achieving optimized BER performance while maintaining fixed data throughput and limited total power.

The remaining part of this paper is organized as follows: Section 2 presents the broadband MIMO system model. The proposed quality criteria including the BER analysis of the investigated MIMO system is reviewed in Section 3. Bit and power loading techniques are developed in Section 4. Simulation results concerning the system performance are obtained and interpreted in Section 5. Finally, Section 6 provides some concluding remarks.

2. Broadband MIMO System Model

In this section the discrete time model for broadband MIMO system is presented. The investigated MIMO system architecture is composed of $n_T$ transmit antennas at the transmitter side and $n_R$ receive antennas at the receiver side. The
frequency selective channel is considered to have \((L_c + 1)\) channel paths, i.e., the effective length of the channel's finite impulse response (FIR). Therefore, the input-output discrete-time formulation for broadband MIMO system is given by

\[
uv[k] = \sum_{\mu=1}^{n_T} \sum_{\kappa=0}^{L_c} h_{\nu,\mu}[k \cdot c_{\mu}[k - \kappa] + n_{\nu}[k],
\]

where \(uv[k]\) is the received symbol at the \(v\)th receive antenna (with \(v = 1, 2, ..., R\)), \(c_{\mu}[k]\) is the transmitted symbol from the \(\mu\)th transmit antenna (with \(\mu = 1, 2, ..., T\)) and \(n_{\nu}[k]\) is the Additive White Gaussian Noise (AWGN) at the \(v\)th receive antenna having zero mean and variance \(U_{R_v}^2\). The parameter \(k\) is the discrete time index. The channel influence between the \(\mu\)th transmit antenna and the \(v\)th receive antenna is represented by the channel coefficients \(h_{\nu,\mu}[k]\) (with \(k = 0, 1, ..., L_c\)), i.e., the channel weightings arising at the \(k\)th channel paths. The FIR of the channel, denoted by the channel coefficients, includes the effect of transmit and receive filtering as well as the multipath component introduced by the channel [9]. Additionally, throughout this paper, the channel coefficients are assumed to undergo a Rayleigh distribution and have the same average power.

In the following analysis, block data transmission model is employed, by which \(K\) data symbols are grouped to form a data block. Afterwards, the data blocks are separated by synchronization symbols. Furthermore, a block fading model is applied, i.e., the channel is assumed to be time-invariant for the duration of a complete data block. By expanding (1) to take into account \((K + L_c)\) consecutive received symbols at each receive antenna, i.e., applying the block data transmission model, the block-oriented model of the broadband MIMO system results in

\[
uv = \sum_{\mu=1}^{n_T} h_{\nu,\mu} \cdot c_{\mu} + n_{\nu},
\]

where \(uv\) is the \((K + L_c) \times 1\) received symbol vector at the \(v\)th receive antenna and is defined as

\[
uv = \begin{bmatrix} u_{\nu}[1], & u_{\nu}[2], & \ldots, & u_{\nu}[K + L_c] \end{bmatrix}^T.
\]

The \((K + L_c) \times K\) channel matrix \(h_{\nu,\mu}\) is the SISO channel influence between the \(\mu\)th transmit antenna and the \(v\)th receive antenna through the complete data block. Considering (1), the channel matrix \(h_{\nu,\mu}\) is according to

\[
H_{\nu,\mu} = \begin{bmatrix}
h_{\nu,[0]} & 0 & \cdots & 0 & 0 \\
h_{\nu,[1]} & h_{\nu,[0]} & \vdots & \vdots & \vdots \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
h_{\nu,[L_c]} & \vdots & \vdots & h_{\nu,[0]} & 0 \\
0 & h_{\nu,[1]} & \cdots & h_{\nu,[1]} & h_{\nu,[0]} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & \cdots & 0 & h_{\nu,[L_c]} & \vdots \\
0 & \cdots & \cdots & 0 & h_{\nu,[L_c]} \\
\end{bmatrix}
\]

The \((K \times 1)\) vector \(c_{\mu}\) denotes the transmit symbol vector from the \(\mu\)th transmit antenna, i.e., the transmit data block and is modeled by

\[
c_{\mu} = \begin{bmatrix} c_{\mu}[1], & c_{\mu}[2], & \ldots, & c_{\mu}[K] \end{bmatrix}^T.
\]

The \((K + L_c) \times 1\) vector \(n_{\nu}\) represents the AWGN at the \(v\)th receive antenna and is given by

\[
n_{\nu} = \begin{bmatrix} n_{\nu}[1], & n_{\nu}[2], & \ldots, & n_{\nu}[K + L_c] \end{bmatrix}^T.
\]

The block-oriented system model described in (2) is expanded to include \(n_R\) receive antennas at the receiver side and results in

\[
\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{n_R} \end{bmatrix} = \begin{bmatrix} H_{1,1} & \cdots & H_{1,n_T} \\ \vdots & \ddots & \vdots \\ H_{n_R,1} & \cdots & H_{n_R,n_T} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_{n_T} \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_{n_R} \end{bmatrix},
\]

which can be written as

\[
u = H \cdot c + n.
\]

The channel matrices \(h_{\nu,\mu}\) introduce ISI between neighboring symbols as well as the interference introduced by the different antenna data streams. To eliminate this interference, as investigated in the literature [6, 7], a popular signal processing technique based on SVD is applied, by which the overall MIMO channel matrix is written as

\[
H = SV \cdot D^{\frac{1}{2}};
\]

Such that \(S\) and \(D\) are unitary matrices composed of the eigenvectors of \(HH^H\) and \(H^H\) respectively, with \((\cdot)^H\) denoting the conjugate transpose operator. The matrix \(V\) is a rectangular diagonal matrix having \((\sqrt{k_1}, \sqrt{k_2}, ..., \sqrt{k_{n_S}})\) on its main diagonal arranged in descending order (with \(n_S \leq K \cdot \min(n_{R}, n_{T})\) per MIMO data block). The singular values \(\sqrt{k_i}\) (with \(i = 1, 2, ..., n_S\)) are the square roots of the \(i\)th positive eigenvalues of \(HH^H\) or \(H^H\). Applying SVD on the system requires signal preprocessing at the transmitter side as well as signal post processing at the receiver side. Such that at the transmitter the MIMO transmitted data block is multiplied by \(D\) and at the receiver the MIMO received data block is multiplied by \(S^{\frac{1}{2}}\). Upon applying SVD signal
processing, the block-oriented system model results in
\[
y = S^H \cdot (H \cdot D \cdot c + n) = S^H \cdot S \cdot V \cdot D^H \cdot D \cdot c + S^H \cdot n
\]
\[
y = V \cdot c + w.
\]

The vector \( y \) denotes the received symbol vector including all \( n_R \) receive antennas after applying SVD, the vector \( c \) is the transmit symbol vector from all \( n_T \) transmit antennas and the vector \( w \) is the AWGN vector at all \( n_R \) receive antennas multiplied by \( S^H \). Upon applying SVD signal processing, the off-diagonal elements in the overall channel matrix \( H \) are eliminated, thus the inter-antenna interference as well as the ISI are eliminated. Moreover, the overall MIMO channel matrix \( H \) is converted into independent non-interfering layers of unequal weightings. This results in different noise immunity per each MIMO layer which requires appropriate handling. Thus to minimize the overall BER for a given fixed data rate, employing layer-specific bit and power allocation schemes is recommended.

### 3. Quality Criteria

The BER for \( M \)-ary Quadrature Amplitude Modulation (QAM) systems over AWGN channel, according to [7, 10], is calculated as follows

\[
P_{BER} = \frac{2}{\log_2(M)} \left(1 - \frac{1}{\sqrt{M}}\right) \text{erfc} \left(\frac{p}{\sqrt{2}}\right),
\]

where \( M \) is the number of signal points in QAM constellation. The parameter \( p \) denotes the signal-to-noise ratio (SNR) and is defined as \( p = \frac{U_A^2}{U_R^2} \), such that \( U_A \) is the half vertical eye opening and \( U_R \) is the noise power per quadrature component. Upon introducing the proposed SVD-equalized MIMO system model, the layer-specific half vertical eye opening at time slot \( k \) (with \( k = 1, 2, \ldots, K \)) results in

\[
U_A^{(\ell,k)} = \sqrt{\xi_{\ell,k}} \cdot U_{S,\ell},
\]

where \( U_{S,\ell} \) is the half level transmit amplitude for layer \( \ell \) (with \( \ell = 1, 2, \ldots, L \)), and the parameter \( L \) describes the number of active MIMO layers, i.e., \( L \leq \min(n_T, n_R) \) for parallel transmission. The singular value \( \sqrt{\xi_{\ell,k}} \) represents the gain of the \( \ell \)th MIMO layer at time slot \( k \). The average power in \( M \)-ary QAM constellation is written in terms of the half level transmit amplitude as follows [7, 10]

\[
P_{s,\ell} = \frac{2}{3} U_{S,\ell}^2 (M - 1),
\]

where \( P_{s,\ell} \) is the average assigned power in \( M \)-ary QAM constellation to the \( \ell \)th MIMO layer. Assuming the total power \( P_s \) is divided equally on all active layers \( L \), the power allocated to each MIMO layer \( P_{s,\ell} \) results in

\[
P_{s,\ell} = \frac{P_s}{L},
\]

where the total transmit power \( P_s \) provided to the MIMO system is defined as

\[
P_s = \sum_{\ell=1}^{L} P_{s,\ell}.
\]

Combining (11), (12) and (13) the layer-specific SNR is according to

\[
\rho^{(\ell,k)} = \frac{\xi_{\ell,k} \cdot U_{S,\ell}^2}{U_R^2} = \frac{3 \cdot \xi_{\ell,k} \cdot P_s}{2 L (M - 1) U_R^2}.
\]

Given (10) and (15), the BER for the \( \ell \)th MIMO layer at time slot \( k \) results in

\[
P_{BER}^{(\ell,k)} = \frac{2}{\log_2(M_{\ell})} \text{erfc} \left(\frac{3 \cdot \xi_{\ell,k} \cdot P_s}{4 L (M - 1) U_{R,\ell}^2}\right).
\]

Taking into account all active MIMO layers, the average BER at time slot \( k \) is obtained as

\[
P_{BER}^{(k)} = \frac{1}{\sum_{\ell=1}^{L} \log_2(M_{\ell})} \sum_{\ell=1}^{L} \log_2(M_{\ell}) \cdot P_{BER}^{(\ell,k)}.
\]

Finally the average BER per MIMO data block \( P_{BER} \) is according to

\[
P_{BER} = \frac{1}{K} \sum_{k=1}^{K} P_{BER}^{(k)}.
\]

By taking different channel SNR into account, the overall BER performance can be evaluated. As described in (17) and (18), the average BER per complete data block \( P_{BER} \) is dominated by the transmission mode employed in each of the active MIMO layers, i.e., the assigned QAM constellations. Therefore, different transmission modes results in different BERs. The transmission modes tabulated in Tab. 1 for \((4 \times 4)\) MIMO systems with fixed data throughput of 8 bit/s/Hz are investigated. It is required to find out the best combination of QAM modes and number of activated MIMO layers which optimizes the BER performance.
4. Bit and Power Loading Techniques

Considering that the average BER at each time slot is obtained by the BER performance of each of the active MIMO layers. Thus, Power Allocation (PA) strategy can be employed to enhance the overall BER performance, such that an appropriate power level is assigned to each active MIMO layer. The PA strategy multiplies the layer-specific half level transmit amplitude $U_{sf \ell}$ with a time-varying layer-specific factor $\sqrt{p_{\ell,i}}$. The aim of the upcoming analysis is to derive the values of $\sqrt{p_{\ell,i}}$ which fulfill the main goal of the PA strategy, i.e., optimal or nearly optimal BER performance. The calculation of the optimal values of $\sqrt{p_{\ell,i}}$, which results in the optimal BER performance, involves using Lagrange multiplier method [6, 11]. Thus excessive complexity optimization problem is introduced. Therefore, sub-optimal PA strategies with lower complexities are of common interest [7, 8, 11]. The layer with the lowest SNR provides the worst impact on the average BER. Therefore, a natural choice is to assign different power levels to each active MIMO layer in a way that an equal-SNR in all active layers is guaranteed at each time slot. Upon applying equal-SNR PA strategy, the half vertical eye opening results in

$$U_{APA}^{(\ell,k)} = \sqrt{p_{\ell,k}} \cdot \sqrt{\xi_{\ell,k}} \cdot U_{sf \ell}.$$ \hspace{1cm} (19)

The layer-specific SNR after the PA strategy is obtained as

$$\rho_{PA}^{(\ell,k)} = \left(\frac{U_{APA}^{(\ell,k)}}{U_{R}^{2}}\right)^{2} = p_{\ell,k} \cdot \rho_{\ell,k}^{(\ell,k)},$$ \hspace{1cm} (20)

where $\rho_{PA}^{(\ell,k)}$ is guaranteed to be constant for $\ell = 1, 2, ..., L$. The total transmit power restriction in the PA strategy is given by

$$P_{s} = \sum_{\ell=1}^{L} P_{s \ell} \cdot p_{\ell,k} = \frac{P_{s}}{L} \sum_{\ell=1}^{L} p_{\ell,k}.$$ \hspace{1cm} (21)

Given (15), (20) and (21), it can be shown that the time-varying layer-specific PA factors are calculated as follows [6]

$$p_{\ell,k} = \frac{L (M_{c} - 1)}{\xi_{\ell,k} \sum_{\nu=1}^{L} (M_{\nu} - 1)}.$$ \hspace{1cm} (22)

The updated layer-specific SNR results in

$$\rho_{PA}^{(\ell,k)} = \frac{3 P_{s}}{2 U_{R}^{2}} \frac{1}{\sum_{\nu=1}^{L} (M_{\nu} - 1)}.$$ \hspace{1cm} (23)

Regardless of the channel quality or the QAM constellation employed, the layer-specific SNRs of all the activated MIMO layers are guaranteed to be equal in (23). Considering (10), (15) and (20), the BER for the $\ell$th MIMO layer at time slot $k$ after the proposed PA strategy is obtained as

$$P_{BER}^{(\ell,k)} = \frac{2 \left(1 - \frac{1}{\sqrt{M_{c}}}ight) \text{erfc} \left(\frac{3 \cdot p_{\ell,k} \cdot \xi_{\ell,k} P_{s}}{4 L (M_{c} - 1) U_{R}^{2}}\right)}{\log_{2}(M_{c})}.$$ \hspace{1cm} (24)

Taking into account (17) and (18), the average BER per MIMO data block $P_{BER}$ after PA is calculated. So far the efficiency of adapting the transmit power level for fixed transmission modes is studied. However in order to achieve further enhancements in the BER performance, additional transmission parameters can be adapted to the varying channel state information, e.g., the modulation mode, the transmission rate, the coding schemes, the constellation size or any combination of these parameters [4, 12]. At the cost of low signaling state information, e.g., the modulation mode, the transmission rate, the coding schemes, the constellation size or any combination of these parameters [4, 12]. At the cost of low signaling state information, e.g., the modulation mode, the transmission rate, the coding schemes, the constellation size or any combination of these parameters [4, 12].

5. Results

In this contribution the efficiency of the TMs listed in Tab. 1 for broadband ($4 \times 4$) MIMO systems with 2 and 5 channel paths is investigated. Fixed data throughput of 8 bits/s/Hz is guaranteed when the predefined TMs introduced in Tab. 1 are employed. Furthermore, the efficiency of PA is studied. The BER curves for the investigated TMs are obtained by computer simulation and depicted in Fig. 1 and Fig. 2. From the simulation results, it can be seen that it is not necessary to activate all MIMO layers in order to minimize the overall BER for a predefined data throughput, e.g., the TM resulting in the best performance at a BER of $10^{-4}$ is
Further enhancements to BER performance are observed by implementing adaptive allocation of transmit power. Additionally, upon analyzing the simulation results, the best TMs at a BER of $10^{-4}$ are specified and shown in bold in Tab. 1. The performance of adaptive TM scheme is investigated. Allowing low signaling overhead, the BER performance of fixed TM scheme can be improved. An adaptive scheme selects the TM which minimizes the BER at each time slot instead of using fixed TM regardless of the channel quality. As depicted in Fig. 3, the adaptive TM scheme outperforms the fixed scheme. Furthermore, the efficiency of transmitting the QAM constellations shown in bold in Tab. 1 over frequency non-selective channel is studied in comparison with the investigated frequency selective 5-paths channel. As depicted in Fig. 4, it is observed that the BER performance of fixed TMs over frequency selective channels is more favorable. Therefore, multipath propagation is not a limiting factor in broadband MIMO systems.

6. Conclusions

Bit auctioning and power assignment strategies in SVD-equalized broadband $(4 \times 4)$ MIMO systems are investigated in this contribution. It turned out that the system BER performance is substantially affected by the additional degrees of freedom introduced by the adaptive bit and power loading schemes. Additionally, in order to minimize the overall BER, activating all the MIMO layers is not necessarily required. Furthermore, the BER performance of the proposed bit and power loading schemes over frequency non-selective MIMO systems is studied in comparison to broadband MIMO systems. It is found out that for a fixed data throughput, the BER performance over broadband MIMO channels is more encouraging. Thus, the delay-spread of the broadband MIMO channel is beneficial in enhancing MIMO BER performance.

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