Signal Deconvolution of Measured Optical MIMO-Channels

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Abstract
In this contribution a \((2 \times 2)\) multiple input multiple output (MIMO) transmission is studied. Based on a measured multi-mode fibre channel at 1326 nm and 1576 nm operating wavelength, the MIMO specific impulse responses are analysed. For the channel measurements a fibre length of 1.4 km was chosen. Since the measured impulse responses are affected by the input impulse as well as by the noise, the system characteristic, i.e. the impulse response, has to be obtained by signal deconvolution. Unfortunately, the signal deconvolution cannot be performed unambiguously especially in case of noisy data. Therefore, in this work different deconvolution criteria are analysed when calculating the MIMO specific impulse responses.

1. Introduction
The concept of MIMO transmission has been investigated since decades now for both, twisted-pair copper cable transmission, suffering from crosstalk between neighbouring wire pairs [1], as well as for multi-antenna radio systems, where signal interference occurs on the radio interface [2, 3]. In the recent past the concept of MIMO transmission over multi-mode fibres has attracted increasing interest in the optical fibre transmission community, targeting at increased fibre capacity [4-6].

In order to analyse the performance of MIMO in the field of optical communication channels, the underlying MIMO specific impulse responses have to be known. Theoretically, an impulse like a Dirac delta pulse has to be chosen in order to measure the channel impulse responses unaffectedly from the input impulse. In practical systems impulses which are similar to a Dirac delta impulse have proven to be advantageous. However, in many cases, the optical power is no longer sufficient to make the modal structure measurable [7]. Thus, the measured impulse responses are affected by the input impulse. In order to obtain the impulse responses unaffectedly from the input impulse, the measured impulse responses have to be deconvolved by e.g. inverse filtering.

Since the measured impulses are often affected by other kinds of disturbances such as noise, the estimate of the deconvolved signal cannot be carried out unambiguously especially in low signal-to-noise situations. That is why inverse filtering in noisy environments is usually not a good idea.

Here, further design criteria have to be taken into consideration when performing the signal deconvolution of noisy signals. Assuming that the noise is white and Gaussian distributed, an improved estimation of the deconvolved signal in terms of the mean square error (MSE) can be obtained by techniques such as Wiener deconvolution or filtering. In this work suboptimal solutions with lower complexity are investigated. That is why, in this contribution the efficiency of predefined filter functions is studied when minimizing the MSE.

A promising empirical approach for signal deconvolution, in systems which are superimposed with noise, was presented by Gans [8]. However, the question regarding the quality of the deconvolution could not be answered finally and requires substantial further research.

In this work based on channel measurements within a 1.4 km multi-mode fibre, different deconvolution criteria are analysed and compared in terms of quality with respect to the MSE, when calculating the deconvolved MIMO specific impulse responses based on the measured ones.

2. Deconvolution of Signals
A linear time-invariant (LTI) system is defined uniquely by its impulse response or its Fourier transform as the corresponding transfer function.

\[
\begin{align*}
  u_1(t) &= U_0 T_s \delta(t) \\
  g_0(t) \quad \Rightarrow \quad g_0(t) \\
  u_2(t) &= U_0 T_s g_0(t)
\end{align*}
\]

Fig. 1. Identifying a LTI system

For the determination of the impulse response, the system can be stimulated with a weighted Dirac delta impulse \(u_1(t) = U_0 T_s \delta(t)\) as a voltage signal with its spectrum \(U_1(f) = U_0 T_s\). The weighting of the Dirac pulse by \(U_0 T_s\) leads to the same area of the Dirac pulse compared with a non-return-to-zero (NRZ) rectangular pulse with the amplitude \(U_0\) (in V) and the duration \(T_s\) (in s), which is meaningful for comparative considerations. The output signal \(u_2(t) = U_0 T_s g_0(t)\) is in this case consistent with
the impulse response $g_k(t)$, except for a form factor.

For estimating the low-pass filter impulse response $g_k(t)$, an appropriate formed input signal $u_1(t)$ is needed. Unfortunately, a Dirac delta impulse with a frequency independent transfer function cannot be realised practically. In applied systems, impulses which are similar to the Dirac delta impulse, have proven to be advantageous. For the determination of the impulse response in optical transmission systems impulses as specified in [7] have proven to be useful. Additionally, when analysing the impulse response of any practical system, the measured impulse $u_3(t)$ is affected by noise. The resulting transmission system model is now depicted in Fig. 2.

![Diagram](image)

**Fig. 2. Transmission system model; the dotted line represents the components of the practical system**

The measured impulse $u_3(t)$ can be decomposed into two parts, namely, the low-pass filter output signal $u_1(t)$ as well as the noise part $n(t)$, and results in

$$u_3(t) = g_k(t) \ast u_1(t) + n(t).$$  

(1)

In the absence of the noise term, i.e. $n(t) = 0$, the system characteristic $g_k(t)$ can be easily obtained by inverse filtering and results in

$$g_k(t) \rightarrow E_k(f) = \frac{U_3(f)}{U_1(f)}.$$  

(2)

Unfortunately, the measured impulse $u_3(t)$ is affected by the noise term $n(t)$. Under these conditions inverse filtering does not work properly anymore. For improving the quality of the signal deconvolution, the low-pass filter regularisation function $g_r(t)$ is applied and the filtered signal results in

$$u_4(t) = g_k(t) \ast u_1(t) \ast g_r(t) + n(t) \ast g_r(t).$$  

(3)

This filter operation affects both the low-pass filter output signal $u_2(t)$ as well as the noise term $n(t)$: On one hand, a low cutoff frequency of $G_r(f)$ suppresses the noise signal as well as the wanted signal strongly whereas in case of a high cutoff frequency, the noise signal as well as the wanted signal remains nearly unaffected. Both cases do not lead to the best results in terms of the MSE when performing inverse filtering based on the output signal $u_4(t)$.

However, predefined filter functions represent a simple approach when minimizing the MSE. Therefore, the filter parameters have to be carefully selected (e.g. the cutoff frequency). With an appropriate selected regularisation function an estimation of the impulse response $g_k(t)$ is given by

$$\hat{g}_k(t) \rightarrow \hat{G}_k(f) = \frac{U_3(f)}{U_1(f)}. $$  

(4)

Classically, the MSE between the impulse response $g_k(t)$ and the estimated impulse response $\hat{g}_k(t)$ is an indicator for finding the optimal set of parameters for a given regularisation function $g_r(t)$. The MSE can be calculated as

$$F_e = E((g_k(t) - \hat{g}_k(t))^2) \rightarrow \min.$$  

(5)

where $E\{\cdot\}$ denotes the expectation functional.

The goal of the further investigation is to estimate the impulse response $g_k(t)$ based on the measured impulse $u_3(t)$. As stated, the filtering affects the noise signal $n(t)$ as well as the signal $u_2(t)$ differently. Therefore, the parameters of the regularisation function $g_r(t)$ have to be carefully selected. In order to find the set of parameters for a given regularisation function two different optimization criteria are applied. The first criterion tries to minimize the MSE when using a given low-pass filter regularisation function. However, for this solution the impulse response $g_k(t)$ has to be known for identifying the optimal set of parameters for the regularisation function, which is practically impossible. That is why other optimization criteria are in the focus of interest. A promising solution was presented by Gans [8]. Here the root mean square of the deconvolved imaginary part of $\hat{g}_k(t)$ is used for finding the parameters of the regularisation function. This optimization criterion can be expressed as

$$E(\{\text{Im}[\hat{g}_k(t,t')]\})^2 \rightarrow \min.$$  

(6)

In order to analyse the choice of the regularisation function $g_r(t)$, a system with the following parameters is studied. The input signal is selected as follows:

$$u_1(t) = U_5 T_5 \delta(t). $$  

(7)

The transfer function of $g_k(t)$ is carried out as an ideal low-pass filter with

$$g_k(t) = \frac{1}{T_k} \cdot \sin \left( \frac{\lambda}{T_k} \right) \rightarrow G_k(f) = \text{rect}(f T_k), $$  

(8)

with:

$$\sin(x) = \frac{\sin(\pi x)}{\pi x}. $$  

(9)
In order to investigate the characteristics of the regularisation function exemplarily, the parameters of the simulation are chosen as follows: $T_s = 8 \text{ ms}$, $T_s/T_a = 16$, $U_s T_s = 1 \text{ Vs}$. The noise power of the signal $n(t)$ is assumed to be $P_r$. A possible transfer function for the regularisation function was presented by [8]:

$$G_r(f) = \frac{|U_4(f)|^2}{|U_1(f)|^2 + \gamma \cdot |C(f)|^2}, \quad (10)$$

with:

$$|C(f)|^2 = 6 - 8 \cos(2\pi T_a) + 2 \cos(4\pi T_a), \quad (11)$$

where $T_a$ is the sampling period. Assuming $|U_4(f)| = 1$, this transfer function simplifies to

$$G_r(f) = \frac{1}{1 + \gamma \cdot |C(f)|^2}. \quad (12)$$

It basically is a low-pass filter with a parameter $\gamma$ which determines the cutoff frequency (see Fig. 3).

![Fig. 3. Regularisation function with different $\gamma$ values.](image)

If the chosen $\gamma$ value is zero no filtering will occur and the transfer function simplifies to

$$G_r(f) = 1. \quad (13)$$

The results obtained after signal deconvolution are highlighted in Fig. 4-6. The choice of the optimal $\gamma$ when minimizing the MSE is depicted in Fig. 4. In noisy environments with increasing noise power $P_r$, the filter effect increases for minimizing the MSE. Fig. 5 shows the effect of optimal filtering with respect to the MSE. As shown by our simulation results the use of the regularisation function is needed for minimizing the MSE. The estimated impulse responses $\hat{g}_k(t, \gamma)$, filtered with different $\gamma$ values, are highlighted in Fig. 6. Here the necessity of a low-pass filter regularisation function in noisy systems before executing the deconvolution is clearly visible.

![Fig. 4. Choice of optimal $\gamma$ when minimizing the MSE](image)

![Fig. 5. MSE as a function of noise power with different parameters of the regularisation function $g_r(t)$.](image)

![Fig. 6. Estimated impulse response $\hat{g}_k(t)$ with optimal and without filtering assuming a noise power of $P_r = 10^2 \text{V}^2$.](image)

3. Optical MIMO System Model

An optical MIMO system can be formed by feeding different sources of light into the fibre, which activates different optical mode groups. Theoretically, it can be done by using two single mode fibres as shown in Fig. 7. The different sources of light lead to different power distribution patterns at the fibre end depending on the
transmitter side light launch conditions. Fig. 8 highlights the measured mean power distribution pattern at the end of a 1.4 km multi-mode fibre.

![Fig. 7. Transmitter side configuration with centre and offset light launch condition](image)

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![Fig. 8. Measured mean power distribution pattern as a function of the light launch position (left: eccentricity \(\delta = 0 \mu m\), right: eccentricity \(\delta = 18 \mu m\)); the dotted line represents the \(50 \mu m\) core size.](image)

Fig. 8. Measured mean power distribution pattern as a function of the light launch position (left: eccentricity \(\delta = 0 \mu m\), right: eccentricity \(\delta = 18 \mu m\)); the dotted line represents the \(50 \mu m\) core size.

Together with the appropriate receiver side spatial configurations, i.e. spot and ring filter (see Fig. 9), the electrical MIMO channel can be formed. Fig. 9 illustrates the corresponding transmitter and receiver side configuration. The resulting electrical MIMO system model is highlighted in Fig. 10.

![Fig. 9. Forming the optical (2x2) MIMO channel (left: light launch positions at the transmitter side with a given eccentricity \(\delta\), right: spatial configuration at the receiver side as a function of the mask diameter \(r\)).](image)

Fig. 9. Forming the optical (2x2) MIMO channel (left: light launch positions at the transmitter side with a given eccentricity \(\delta\), right: spatial configuration at the receiver side as a function of the mask diameter \(r\)).

4. Results

Investigations in [9] have shown that an eccentricity of \(\delta = 10 \mu m\) and a mask diameter of \(r = 15 \mu m\) were found to be beneficial for minimizing the overall bit-error rate at a fixed data rate. Fig. 11 highlights the input impulses for measuring the MIMO-specific impulse responses. The pulses are chosen in a way that the same optical power is coupled into the multi-mode fibre core. Theoretically, an impulse similar to a Dirac delta pulse should be chosen in order to measure the channel impulse response unaffectedly from the input impulse. The chosen input impulses shown in Fig. 11 are a good compromise to a signal like a Dirac delta impulse with a reasonable amount of coupled optical transmit power.

![Fig. 10. Electrical MIMO system model (example: \(n = 2\)).](image)

Fig. 10. Electrical MIMO system model (example: \(n = 2\)).

![Fig. 11. Input impulse for calculating the MIMO-specific impulse responses at different operating wavelength](image)

Fig. 11. Input impulse for calculating the MIMO-specific impulse responses at different operating wavelength.

For analysing the effect of the regularisation function, the input signal depicted in Fig. 11 with an operating wavelength of \(\lambda = 1576\) nm is used and the following parameters are chosen: \(T_s = 200\) ps, \(T_b/T_s = 50\). The used impulse response \(g_k(t)\) is described in (8). Using the MSE criterion, the obtained quality of the signal deconvolution is shown in Fig. 13 with the corresponding \(\gamma\) values depicted in Fig. 12. As stated before, the \(\gamma\) values increase with increasing noise power \(P_n\). Comparing the quality of the deconvolution, i.e. \(F_B(P_n, \gamma)\), the MSE criterion with the known \(g_k(t)\) shows a superior behaviour compared with the Gans’ criterion (6). However, the loss introduced by the Gans’ criterion is quite acceptable over a wide range of noise power, which is added to the system.

The MIMO-specific impulse responses are obtained after filtering measured impulse responses with the regularisation function \(g_k(t)\) at a fixed \(\gamma\)-
value of 0.05. The results after signal deconvolution are depicted in Fig. 14 and Fig. 15 and show the influence of the modal as well as chromatic dispersion of the 1.4 km multi-mode fibre.

5. Conclusions

In this work, the effect of signal deconvolution of noisy data has been analysed. Next to optimal solutions such as Wiener filtering in this contribution the efficiency of predefined filter functions has been studied. Our results obtained by computer simulation show that the quality of deconvolved signals can be improved when using properly selected regularisation filter functions. Here, the Gans' criterion has proven to be beneficial in practical measurement campaigns. As an application, the signal deconvolution has been applied to measured optical MIMO impulse responses.

Bibliography


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