Power Allocation Techniques in MIMO Systems

Francisco Cano-Broncano, Universidad Politécnica de Madrid
Andreas Ahrens, Wismar University
César Benavente-Peces, Universidad Politécnica de Madrid

Abstract

Bit- and power allocation techniques for Multiple-Input Multiple-Output (MIMO) systems are investigated for minimizing the overall bit-error rate (BER) at a fixed data rate. In this work optimal power allocation (PA) technique based on the Lagrange multiplier method is studied and compared with suboptimal solutions. Our results obtained by computer simulations show that power allocation works very well in combination with an appropriate number of activated MIMO.

1. Introduction

The use of multiple antennas at the transmitter and multiple antennas at the receiver side, also known as a MIMO system, improves the link reliability of wireless communications, i.e. reduces the bit-error probability by exploiting received multipath signals without increasing the required transmitted power and bandwidth [1]. In order to use the available MIMO resources in a nearly optimum way, bit and power allocation strategies are needed. Next to bit allocation, power allocation can be used to balance the bit-error probabilities in the number of activated MIMO layers [2]. Provided perfect channel state information (PCSI) is available at the transmitter side two major optimization problems are considered to be solved: firstly, the bit loading and secondly, the power allocation optimization problem [3]. Bit and power allocation algorithms, which can improve the MIMO system performance significantly, have received a huge research activity lately, and can be divided into two groups according to their performance: optimal and suboptimal algorithms. Optimal allocation algorithms usually have high computational complexity, making them difficult to apply to practical communication systems [4]. In order to implement bit and power allocation in practical communication systems computationally efficient allocation algorithms are in the focus of interest. In this work besides the bit allocation problem different PA strategies are studied.

2. MIMO System Model

According to [2] a frequency non-selective MIMO communication link with $n_T$ antennas in transmission and $n_R$ in reception can be described as

$$u = H \cdot c + n,$$

where $u$ corresponds to the $(n_R \times 1)$ receive vector, $H$ is the $(n_R \times n_T)$ channel matrix, $c$ is the $(n_T \times 1)$ transmit vector and $n$ is the vector of the Additive White Gaussian Noise. Furthermore, it is assumed that the coefficients of the $(n_R \times n_T)$ channel matrix $H$ are independently Rayleigh distributed with equal variance. Details on the SVD-based MIMO transmission model are given in [5].

Using Singular Value Decomposition (SVD) the resulting layer specific system model is proposed in Fig. 1 with different eye openings per activated MIMO layer $\ell$ and per transmitted symbol block $k$ according to

$$U_A^{(\ell,k)} = \sqrt{\xi_{\ell, k}} \cdot U_s \ell,$$

with $\ell = 1, \ldots, L$ and $L \leq \min(n_T, n_R)$.

Considering QAM constellations, the average transmit power $P_s \ell$ per MIMO layer may be expressed as

$$P_s \ell = \frac{2}{3} U_s^2 \ell (M_T - 1).$$

By taking $L \leq \min(n_T, n_R)$ MIMO activated layers into account, the overall transmit power results in...
The layer-specific bit-error probability at time slot $k$ is obtained as [5]

$$\Pr_b^{(k)} = \frac{2}{\log_2(M_t)} \left(1 - \frac{1}{\sqrt{M_t}}\right) \text{erfc}\left(\frac{U_{PA}^{(k)}}{\sqrt{2}U_R}\right).$$

The aggregate bit-error probability at time slot $k$, taking $L$ activated MIMO-layers into account, results in

$$P_b^{(k)} = \frac{1}{\sum_{\ell=1}^{L} \log_2(M_{\ell})} \sum_{\ell=1}^{L} \log_2(M_{\ell}) P_b^{(k)}. \quad (6)$$

Finally, the BER of the whole system is obtained by considering the different transmission block SNRs.

### 3. Bit and Power Allocation

Thanks to bit and power allocation techniques it is possible to use the wireless channel in an optimized way, i.e. minimizing the BER performance at a fixed data rate under the constraint of a limited total MIMO transmit power. In general, the BER performance regarding the channel quality is affected by both the layer-specific weighting factors $\sqrt{\xi_{\ell,k}}$ and the QAM-constellation sizes $M_{\ell}$. Assuming a fixed data rate regardless the channel quality the resulting layer-specific QAM constellations for a fixed data rate under the constraint of a limited total MIMO transmit power. In general, the BER performance regarding the channel quality is affected by both the layer-specific weighting factors $\sqrt{\xi_{\ell,k}}$ and the QAM-constellation sizes $M_{\ell}$. Assuming a fixed data rate regardless the channel quality the resulting layer-specific QAM constellations for a fixed data rate under the constraint of a limited total MIMO transmit power.

Tab. 1. Parameters for bit loading: Investigated QAM transmission modes for fixed transmission bit rate.

<table>
<thead>
<tr>
<th>throughput</th>
<th>layer 1</th>
<th>layer 2</th>
<th>layer 3</th>
<th>layer 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 bit/sHz</td>
<td>256</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8 bit/sHz</td>
<td>64</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8 bit/sHz</td>
<td>16</td>
<td>16</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8 bit/sHz</td>
<td>16</td>
<td>4</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>8 bit/sHz</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

Fig. 2. SVD-based layer-specific transmission model per MIMO layer $\ell$ and per transmitted data block $k$ including MIMO-layer PA

Following the allocation of the bits per layer, PA can be added to optimize the overall BER. The layer-specific PA factors $\sqrt{P_{\ell,k}}$ adjust the half-vertical eye opening per symbol block to (Fig.2)

$$U_{PA}^{(k)} = \sqrt{P_{\ell,k} \cdot \sqrt{\xi_{\ell,k}} \cdot U_s}. \quad (7)$$

Resulting in the layer-specific transmit power per symbol block $k$

$$P_{s,PA}^{(k)} = P_{\ell,k} \cdot P_{s,\ell}. \quad (8)$$

where $P_{s,\ell}$ denotes the allocated power per MIMO layer without PA i.e., $P_{s,\ell} = P_s / L$. Therein the parameter $L$ describes the number of activated MIMO layers with $L \leq \min(n_R, n_P)$. Taking all activated MIMO layers into account, the overall transmit power per symbol block $k$ is obtained as

$$P_{s,PA} = \sum_{\ell=1}^{L} P_{s,PA}^{(k)}. \quad (9)$$

Particularizing (5) with (7) deals to the layer-specific bit-error probability at the time $k$ given by

$$p_{b,PA}^{(k)} = \frac{2}{\log_2(M_t)} \left(1 - \frac{1}{\sqrt{M_t}}\right) \text{erfc}\left(\frac{U_{PA}^{(k)}}{\sqrt{2}U_R}\right). \quad (10)$$

In order to find the optimal set of PA parameters minimizing the overall BER, i.e., $\sqrt{P_{\ell,k}}$, the Lagrange multiplier method is used. Using the Lagrange multiplier method, the cost function $J(p_0, \ldots, p_{N_R - 1})$ may be expressed as

$$J(\ldots) = \frac{1}{\sum_{\ell=1}^{L} \log_2(M_{\ell})} \sum_{\ell=1}^{L} \log_2(M_{\ell}) P_{b,PA}^{(k)} + \lambda \cdot B. \quad (11)$$

where $\lambda$ denotes the Lagrange multiplier. The parameter $B$ in (11) describes the boundary condition

$$B = \sum_{\ell=1}^{L} \left(P_{s,\ell} - P_{s,PA}^{(k)} \right) = 0 \quad (12)$$

$$= \sum_{\ell=1}^{L} P_{s,\ell} (1 - p_{\ell,k}) = 0. \quad (13)$$

A natural choice is again to opt for a scheme that uniformly distributes the overall transmit-power along the number of activated MIMO layers, i.e. $P_{s,\ell} = P_s / L$. In this case, the boundary condition simplifies to

$$B = \frac{P_s}{L} \sum_{\ell=1}^{L} (1 - p_{\ell,k}) = 0. \quad (14)$$

Following (14) the transmit power coefficients have to fulfill the following equation $\sum_{\ell=1}^{L} P_{\ell,k} = L$. Differentiating the Lagrangian cost function
\[ J(p_{1,k}, p_{2,k}, \ldots, p_{L,k}) \] with respect to the \( p_{\ell,k} \) and setting it to zero, leads to the optimal set of PA parameters.

### Tab. 2. Investigated channel profiles assuming a (4 × 4) MIMO system

<table>
<thead>
<tr>
<th>Profile</th>
<th>layer 1</th>
<th>layer 2</th>
<th>layer 3</th>
<th>layer 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>CM-1</td>
<td>1.7500</td>
<td>0.8750</td>
<td>0.4375</td>
<td>0.2188</td>
</tr>
<tr>
<td>CM-2</td>
<td>1.9000</td>
<td>0.6333</td>
<td>0.2111</td>
<td>0.0704</td>
</tr>
</tbody>
</table>

\[
(15)
\]

Since the optimal PA solution is notably computationally complex to implement, a suboptimal solution is investigated, which concentrates on the argument of the complementary error function. In this particular case the argument of the complementary error function

\[
(16)
\]

Here, for each symbol in the transmitted MIMO symbol vector the same half vertical eye opening of

\[
(17)
\]

can be guaranteed (\( \ell = 1, \ldots, L \)), i.e.,

\[
(18)
\]

When assuming an identical detector input noise power for each channel output symbol, the above-mentioned equal quality scenario is encountered.

### 4. Results

In order to study the effect of PA thoroughly, two different fixed channel profiles as shown in Tab. 2 are investigated. For comparison reason, the channel profile CM-1 offers a lower unequal weighting compared to the channel CM-2. The obtained BER curves for both channel profiles are shown in Fig. 3 and Fig. 4. In order to use the MIMO channel in an optimized way not all MIMO layers should be necessarily activated. Furthermore, PA guarantees in combination with an appropriate number of activated MIMO layers the best BER performance when transmitting a fixed data rate of 8 bit/s/Hz over frequency non-selective MIMO channels. From the simulation results it can be seen that not all MIMO layers should be necessary activated in order to get the best BER. In Fig. 5 the obtained BER curves with the optimal PA are composed with the above mentioned equal quality criteria. As demonstrated by computer simulation the loss in the overall BER with the equal quality criteria is quite acceptable when using an optimized bit loading. Fig. 6 shows a comparison of the BER curves among the listed QAM transmission modes in Tab. 1 with and without PA when transmitting 8 bit/s/Hz over frequency non-selective MIMO channels.

### 4. Conclusions

The chance to design how to allocate the bits and the power per active layer affect the overall MIMO system performance remarkably. The investigation described in this work compares the BER curves
among fixed QAM transmission modes and fixed channel profiles considering optimal and suboptimal PA solutions. In this work a noteworthy BER performance improvement has been accomplished by using bit and power allocation techniques over frequency non-selective (4 × 4) MIMO channels. Considering that the loss in the overall BER with the suboptimal equal-SNR PA criteria is quite acceptable compared to the optimal Lagrange multiplier PA method, it is suitable to utilize the suboptimal equal-SNR PA technique for minimizing the overall BER performance due to the lower computational complexity.

![Fig. 5. BER with optimal PA (dotted line), equal-SNR PA (dashed line) and without PA (solid line) when using the transmission modes introduced in Tab. 1 and transmitting 8 bit/s/Hz over channel CM-2](image)

![Fig. 6. BER with equal-SNR PA (dotted line) and without PA (solid line) when using the transmission modes introduced in Tab. 1 and transmitting 8 bit/s/Hz over non-frequency uncorrelated selective MIMO channels](image)

**Bibliography**


**Authors**

Ing. Francisco Cano-Broncano  
Universidad Politécnica de Madrid.  
E. U. I. T. T.  
Carretera de Valencia km 7 28031 Madrid.  
tel. (0034) 91 336 77 80  
email: fcbroncano@gpss.eit.t.upm.es

Prof. Dr.-Ing. habil. Andreas Ahrens  
Hochschule Wismar  
University of Technology, Business and Design  
Philipp-Müller-Strasse 14  
23966 Wismar  
tel. (0049) 3841 7537330  
fax (0049) 3841 7537130  
email: andreas.ahrens@hs-wismar.de

Prof. Dr. César Benavente-Peces  
Universidad Politécnica de Madrid.  
E. U. I. T. T.  
Carretera de Valencia km 7 28031 Madrid.  
tel. (0034) 91 336 77 80  
email: cesar.benavente@upm.es